Deep Generative Models of Gravitational Waveforms via Conditional Autoencoder

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Motivation

In both processes of detection and PE of gravitational wave events, a huge number of theoretical waveform templates are required for matched filtering. However, due to the unavoidable strong gravity regime for the mergers of two compact objects, it is notoriously difficult to calculate the CBC dynamics and the associated gravitational waveforms. To accelerate the generation of theoretical waveforms for practical applications, We aim to construct some deep learning neural network [1, 2] to generate the CBC gravitational waveforms of high accuracy by giving the source parameters.

Data Set

	m_1	m_2	q	Δm	Train	Valid	Test
$q \le 5$	[5.0,75.0]	[5.0,75.0]	[1,5]	0.25	24865	3552	7104
$q > 5$	[5.0,75.0]	[5.0,75.0]	[5,10]	0.25	682	36	2873

- Here the mass ratio is denoted by $q \equiv m_2/m_1$ with $1 \le q \le 10$. The corresponding percentages of (training, validation, test) is (70%, 10%, 20%) for LMR (low-mass-ratio) data ($q \le 5$), and is (19%, 1%, 80%) for HMR (high-mass-ratio) data ($q > 5$).
- Note that the fraction of the HMR templates is **only about** 2.46% of the total training data set, including both training and validation data.

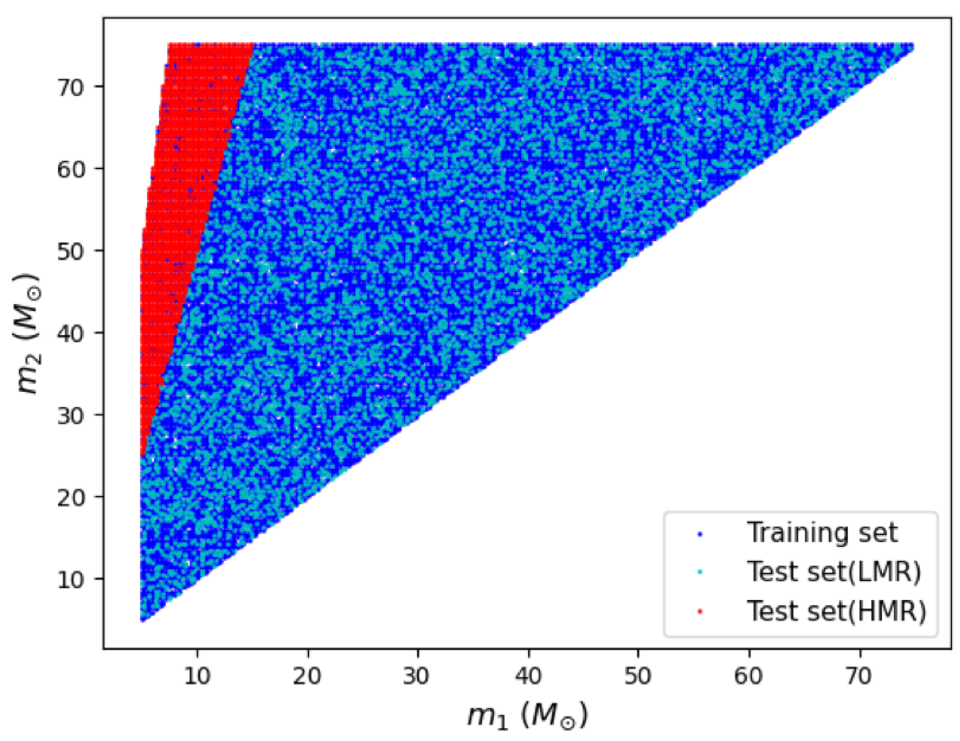

Figure 1: Tomography

Network Structure

- latent vector/space plays the role of the reduced order storing compressed representation.
- Reconstruction Loss compares templated waveform and generated waveform by our model.
- Latent Loss compares the outputs Encoder1 and Encoder2.
- For 2C2E1D model, strain is normalized frequency and amplitude, label is masses of two black hole, key is normalized mean and variance $(\mu_\omega, \sigma_\omega, \mu_A, \sigma_A)$, and target is un-normalized strain.
- After the training, we can remove the waveform input and Encoder2. Then Decoder becomes a machine of generating the waveforms with the given source and Encoder1.

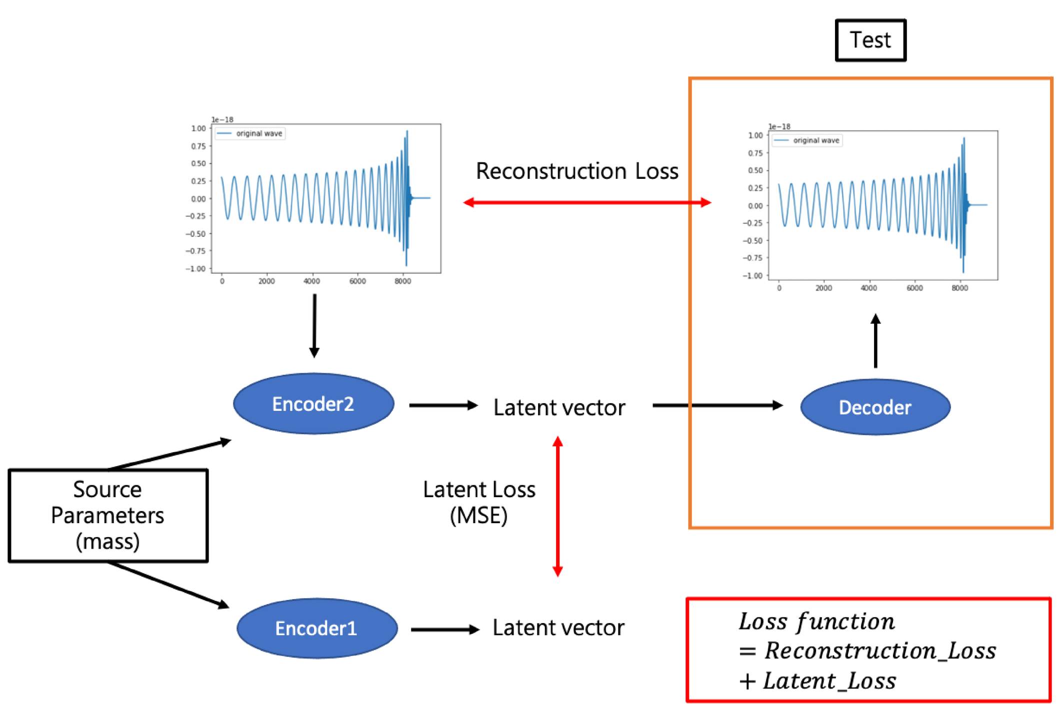

Figure 3: Basic CAE structure

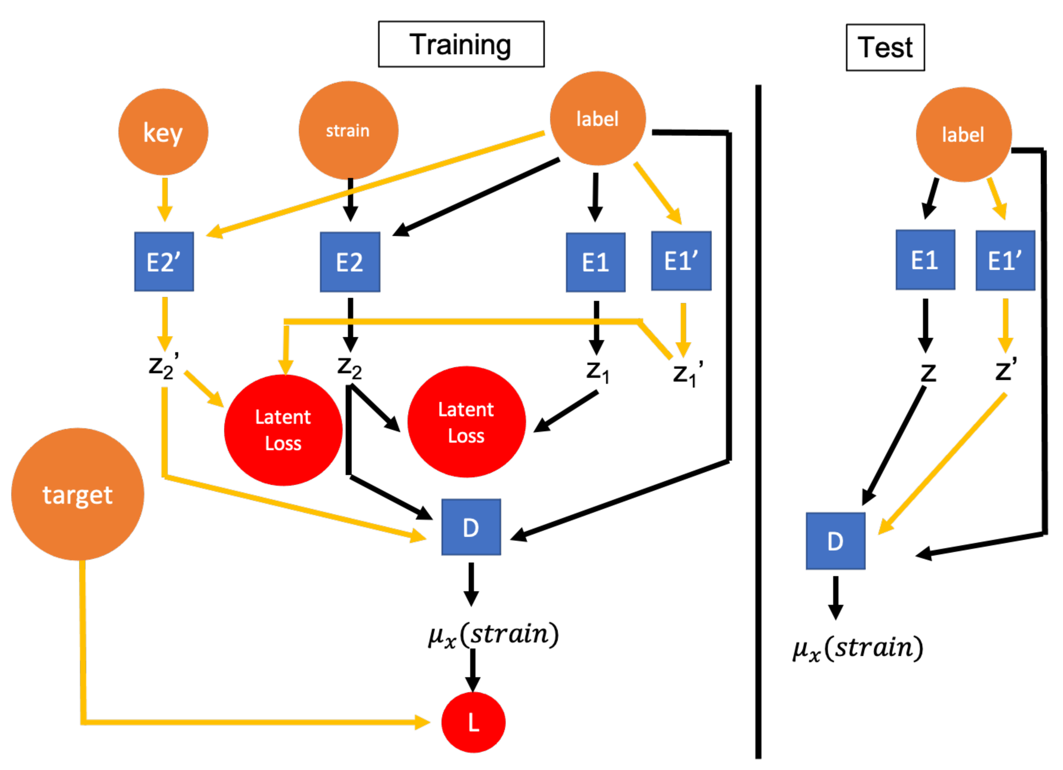

Figure 4: Schematic structure of the 2c2E1D model

Result

	CAE + NN	2CAE	1C2E1D	2C2E1D
Accuracy (LMR)	85.73%	89.92%	97.65%	98.20%
Accuracy (HMR)	55.95%	67.20%	97.70%	97.02%

Figure 5: Average FF for different models

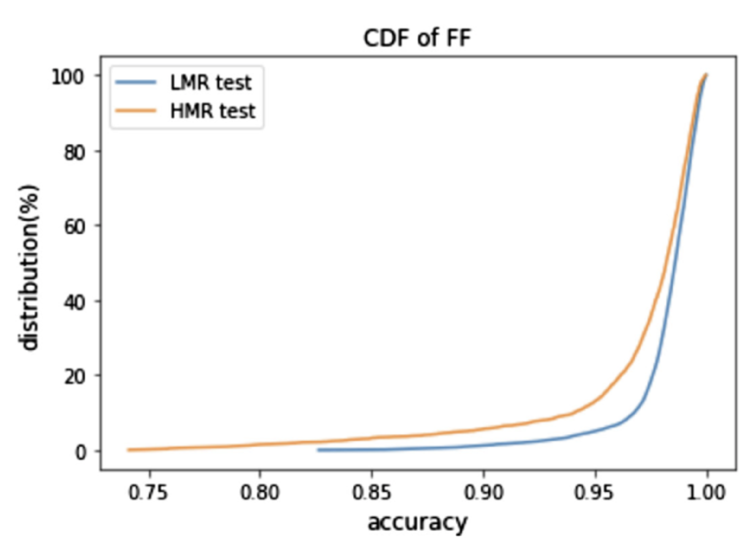

Figure 6: CDF for 2C2E1D FFs

The 2C2E1D model is the best with more than 97% for both the low-mass-ratio and high-mass-ratio waveform generation. This demonstrates the viability of our best waveform model to be implemented in the practical gravitational wave data analysis and parameter estimation (PE). Especially, the generation time (**about 1 millisecond**) of a single waveform is **10 to 100 times faster** than the traditional EOBNR method, and the impressive accuracy for HMR waveform generations is encouraging because fraction of the HMR waveforms in the training and validation data set is less than 3%.

Data Preparation

A strain is the linear combination of the two polarization modes :

$$h(t) = h_+(t) + i h_\times(t)\ . \qquad (1)$$

Instantaneous phase :

$$\theta(t) = \tan^{-1}\left(\frac{h_\times(t)}{h_+(t)}\right) \qquad (2)$$

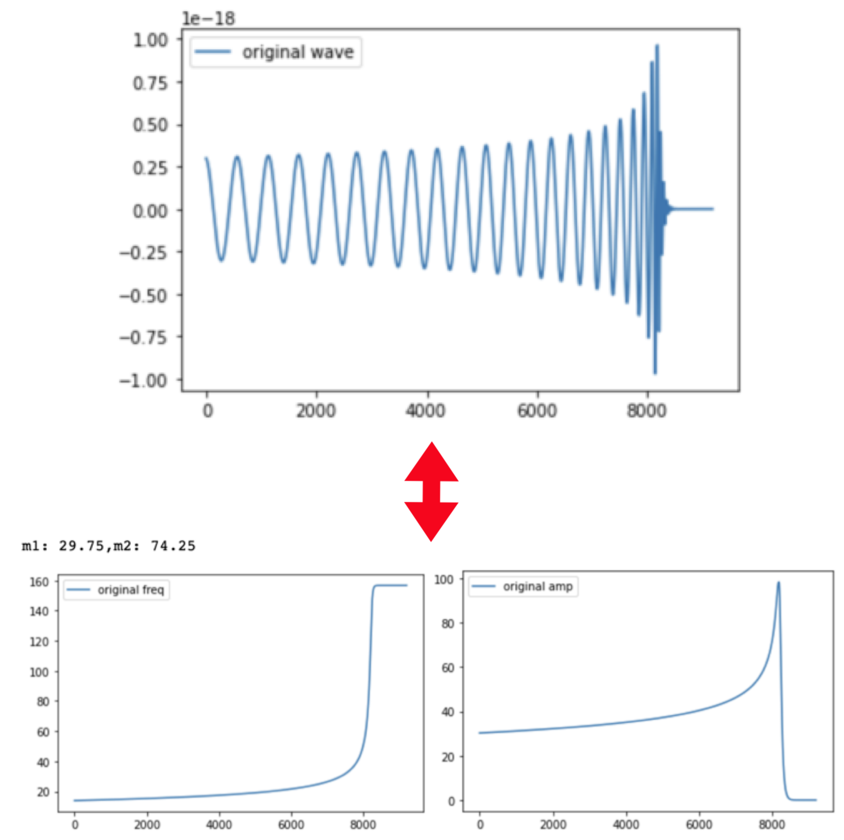

Figure 2: Decomposition of a time series strain

Normalization

The way of normalization we adopt is as follows:

$$\hat{\omega}(t) = \frac{\omega(t) - \mu_\omega}{\sigma_\omega}\ , \qquad \hat{A}(t) = \frac{A(t) - \mu_A}{\sigma_A} \qquad (3)$$

where the normalization parameters $(\mu_\omega, \sigma_\omega)$ are respectively mean and variance evaluated from the 8192 segments of $\omega(t)$, and similarly for (μ_A, σ_A).

Accuracy

Overlap between two waveforms :

$$\langle h_1 | h_2 \rangle = 4\,\Re \int_0^\infty \frac{\tilde{h}_1(f)\tilde{h}_2(f)^*}{S_n(f)} df \qquad (4)$$

where $\tilde{h}_i(f)$ is the Fourier transform of $h_i(t)$ and $S_n(f)$ is the power spectral density (PSD) of detector's noise.

Faithfulness [3] is adopted to compare $h_{\rm ML}(t)$ generated by our waveform model and the standard EOB waveform $h_{\rm EOB}(t)$,

$$\mathrm{FF} = \max_{t_0, \phi_0} \left[\frac{\langle h_{\rm EOB} | h_{\rm ML} \rangle}{\sqrt{\langle h_{\rm EOB} | h_{\rm EOB} \rangle \langle h_{\rm ML} | h_{\rm ML} \rangle}} \right] \qquad (5)$$

where t_0 and ϕ_0 are respectively the initial time and inital phase of $h_{\rm EOB}(t)$

References

[1] G. E. Hinton and R. S. Zemel. Autoencoders, minimumdescription length and helmholtz free energy.

[2] D. P. Kingma and M. Welling. An introduction to vari-ational autoencoders

[3] A. Buonanno, Y. Pan, J. G. Baker, J. Centrella, B. J. Kelly, S. T. McWilliams, and J. R. van Meter, Approaching faithful templates for nonspinning binary black holes using the effective-one-body approach.

[4] PhysRevD.103.124051 (This work)