Mass-spin Re-Parameterization for Rapid PE of Inspiral Gravitational-Wave Signals

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Objectives

The objective of our research is to reduce the time for Bayesian estimation of parameters of inspiral gravitational-wave, keeping the estimation results unchanged.
For this, we introduce an alternative set of mass-spin parameters for Markov chain Monte Carlo(MCMC) sampling.

Methods

The restricted post-Newtonian waveform in 1.5PN can be represented as

$$\tilde{h}(f) = \mathcal{A}\left(\frac{f}{f_{\text{ref}}}\right)^{-\frac{7}{6}} e^{-i\Psi(f)} \tag{1}$$

with the phase function

$$\Psi(f) = \psi^1\left(\frac{f}{f_{\text{ref}}}\right)^{-\frac{5}{3}} + \psi^2\left(\frac{f}{f_{\text{ref}}}\right)^{-1} + \psi^3\left(\frac{f}{f_{\text{ref}}}\right)^{-\frac{2}{3}} + \psi^4 + \psi^5\left(\frac{f}{f_{\text{ref}}}\right). \tag{2}$$

ψ^i is a known function of physical parameters, such as masses and spins. The new parameters are

$$\mu_n \equiv \sum_i U_{ni}\psi^i, \tag{3}$$

where U_{ij} is an orthogonal matrix that diagonalizes the Fisher matrix for ψ^i, $\Gamma_{ij} \equiv (\partial h/\partial\psi^i, \partial h/\partial\psi^j)$.

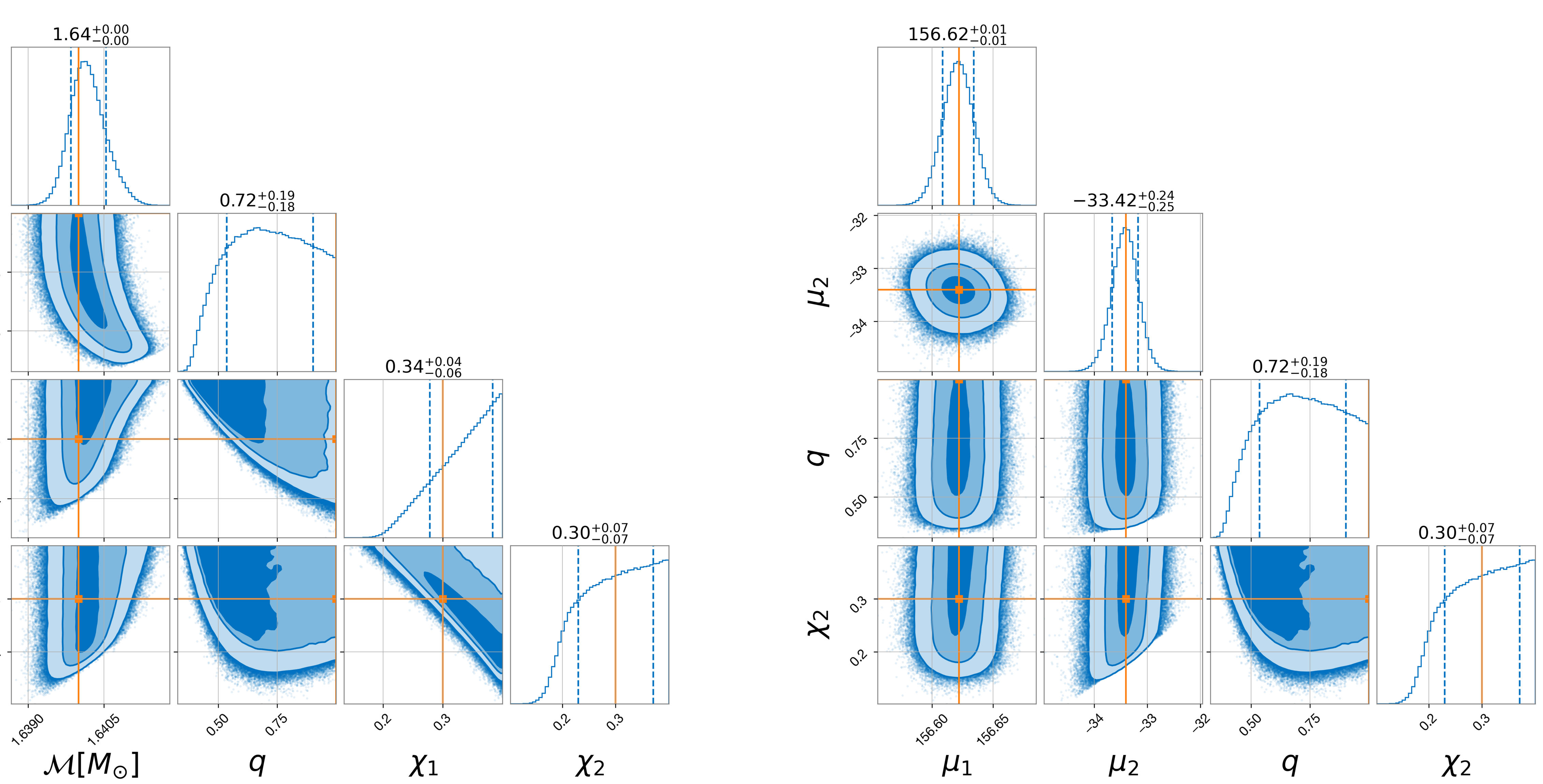

Figure 1: The posterior distribution for the case #2, in $(\mathcal{M}, q, \chi_1, \chi_2)$ space(left) and $(\mu_1, \mu_2, q, \chi_2)$ space(right).

Advantages

Using μ as sampling parameters makes the posterior distribution to be simple in the parameter space. Assuming Gaussian noise, the log-likelihood at $\mu = \hat{\mu} + \Delta\mu$ can be approximated as $-\frac{1}{2}\sum_n \lambda_n \Delta\mu_n^2$. The posterior distribution around the true values in the μ space is

$$p(\mu|d) \propto p(\mu)\prod_n e^{-\frac{1}{2}\lambda_n\Delta\mu_n^2}, \tag{4}$$

where $p(\mu)$ is the prior distribution.
We can generate posterior samples efficiently in μ space, and get the posterior distribution for physical parameters easily by converting the generated samples.

Implementation

Case	Spin prior	$m_1[M_\odot]$	$m_2[M_\odot]$	χ_1	χ_2
#1	Narrow $\lvert\chi_1\rvert, \lvert\chi_2\rvert < 0.05$	1.8839	1.8839	0.02	0.02
#2	Semi-broad $\lvert\chi_1\rvert, \lvert\chi_2\rvert < 0.4$	1.8839	1.8839	0.3	0.3
#3	Broad $\lvert\chi_1\rvert, \lvert\chi_2\rvert < 0.99$	2.2588	1.5811	0.5	0.5

Table 1: Test cases.

We compare the parameter estimation with and without our re-parameterization using Bilby [1] and `PTMCMCSampler` [2]. The test is done in 3 cases of Table 1, which are different in spin prior range.
Without and with re-parameterization, the sampling parameters are $(\mathcal{M}, q, \chi_1, \chi_2, \phi_c, t_c)$ and $(\mu_1, \mu_2, q, \chi_2, \phi_c, t_c)$ respectively.
We assume a single detector and 128s/2048Hz data. Injected and searching waveform is IMRPhenomD approximant. Geometrical parameters are chosen arbitrarily to make SNR to be around 10.

Results

To evaluate the sampling efficiency of MCMC, the maximum integrated auto-correlation time(IAT) is used [3]. Each iteration of MCMC generates a posterior sample, but the samples are auto-correlated. The maximum IAT shows the required number of iterations equivalent to one independent sample.

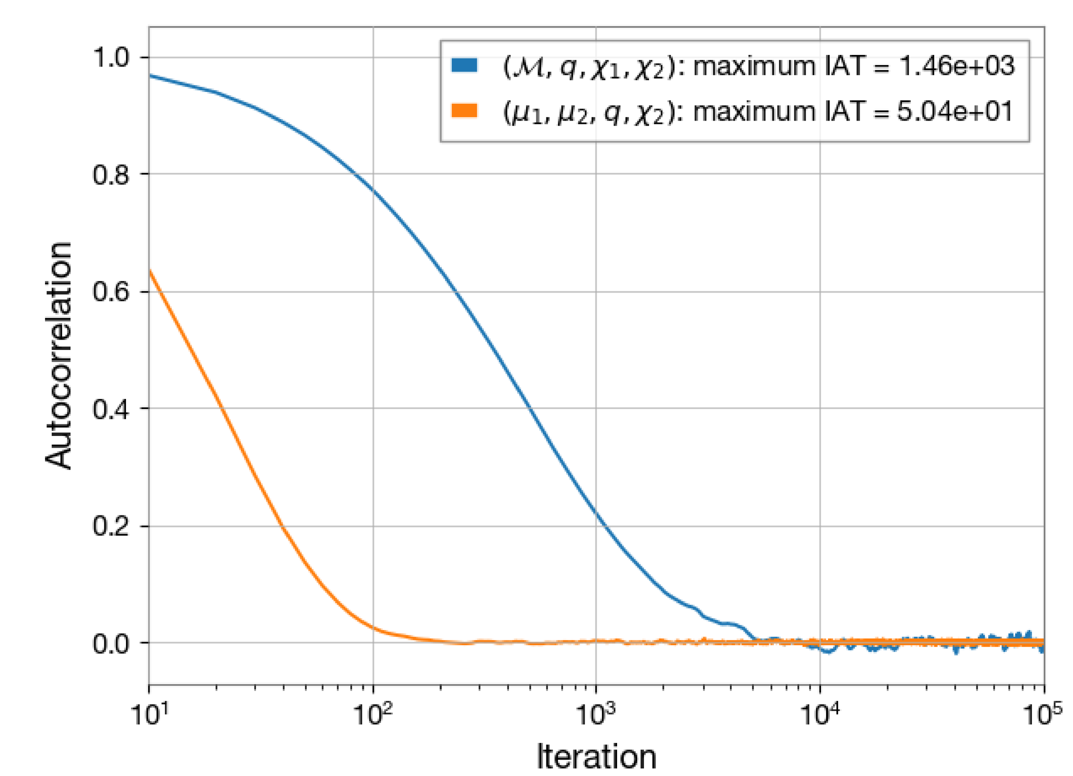

Figure 2: The auto-correlation function of generated samples, with(orange) and without(blue) re-normalization.(Case #2)

Table 2 shows the estimated maximum IATs. In case #2, the maximum IAT is reduced by 1/29, and it implies the reduction of estimation time by 1/29 for the identical estimation precision.

Case	IAT$(\mathcal{M}, q, \chi_1, \chi_2)$	IAT$(\mu_1, \mu_2, q, \chi_2)$	Ratio
#1	427	47.8	8.93
#2	1.46×10^3	50.4	29.0
#3	1.10×10^5	717	153

Table 2: The comparison of maximum IAT values estimated from the samples.

After the sufficient number of iterations, the estimated posterior distributions are well matched.

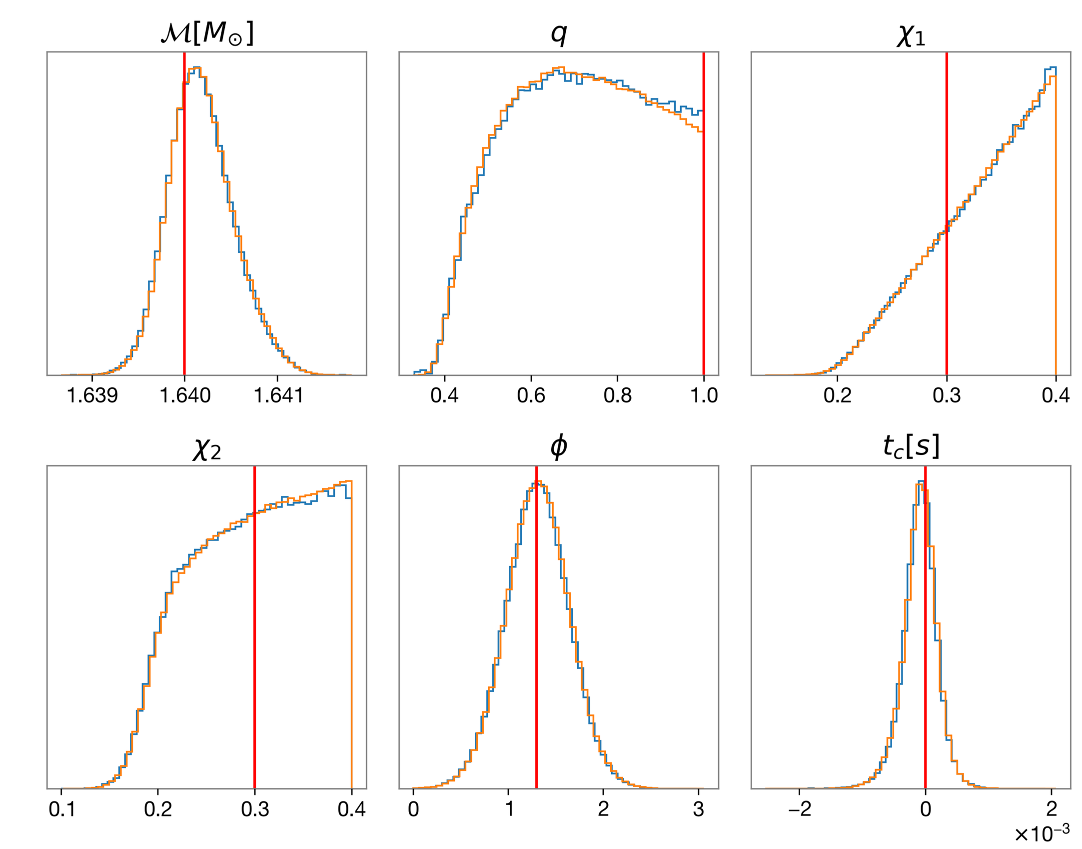

Figure 3: The estimated marginal posteriors, with(orange) and without(blue) re-normalization.(Case #2)

Conclusion

In this work we have introduced a new set of mass-spin parameters for compact binary inspiral waveform, which makes the posterior distribution simple. In all injection tests, the new parameters improved the efficiency of the sampling process. Especially the improvement was remarkable when posterior distribution had complicated shape in the usual mass-spin parameter space, like broader spin prior range cases.

References

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