Modelling frequency domain inspiral-merger-ringdown waveforms for eccentric binary black hole mergers

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Comparison of post-Newtonian waveforms with Numerical Relativity simulations

Amplitude and frequency data for the dominant ℓ=2, m=2 mode from an eccentric Numerical Relativity (NR) simulation (SXS:BBH:1364) is compared with a post-Newtonian (PN) prescription formally coded up in **LALSuite**[1] with the name, **EccentricTD** and is based on the work of Ref. [2].

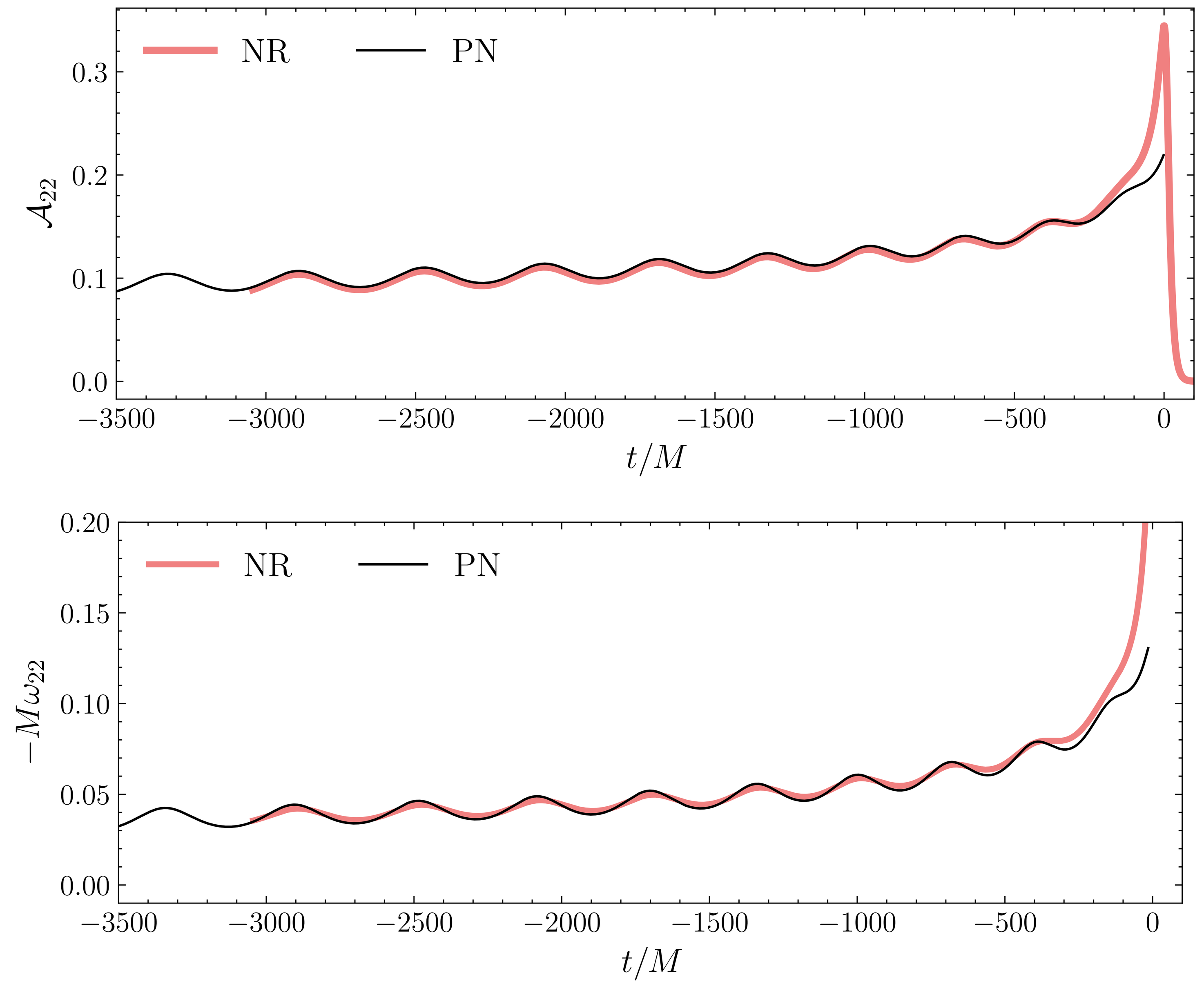

Figure 1. Dominant mode amplitude (top) and frequency (bottom) data from an eccentric Numerical Relativity (NR) simulation (SXS:BBH:1364) together with an Eccentric post-Newtonian (PN) model [2] are plotted.

Close agreements between the PN and NR waveforms in the inspiral part of the signal allow for **hybridisation** and hence the construction of complete **inspiral-merger-ringdown** waveforms in time domain.

Construction of PN-NR hybrids

Complete inspiral-merger-ringdown (**IMR**) waveforms are constructed by matching PN and NR waveforms in a region where the PN prescription closely mimics the NR data.

A least-square minimisation over a time and phase shift parameters (t_0, φ_0) for the difference between the PN and NR waveforms in the matching time interval (t_1, t_2) is performed for constructing the hybrids shown in Figure 2.

$$\delta = \min_{t_0,\varphi_0} \int_{t_1}^{t_2} dt \sum_{\ell,m} \left| h^{\rm NR}_{\ell m}(t-t_0)\, e^{im\varphi_0} - h^{\rm PN}_{\ell m}(t) \right|$$

A Hybrid

Hybrid constructed by matching waveform data for the NR simulation, **SXS:BBH:1364**, and the PN model (**EccentricTD**) evaluated with the same intrinsic binary parameters.

The green dotted line marks the beginning of the NR waveform and the shaded grey region $t \in (1000M, 2000M)$ shows the matching window where hybridization was implemented.

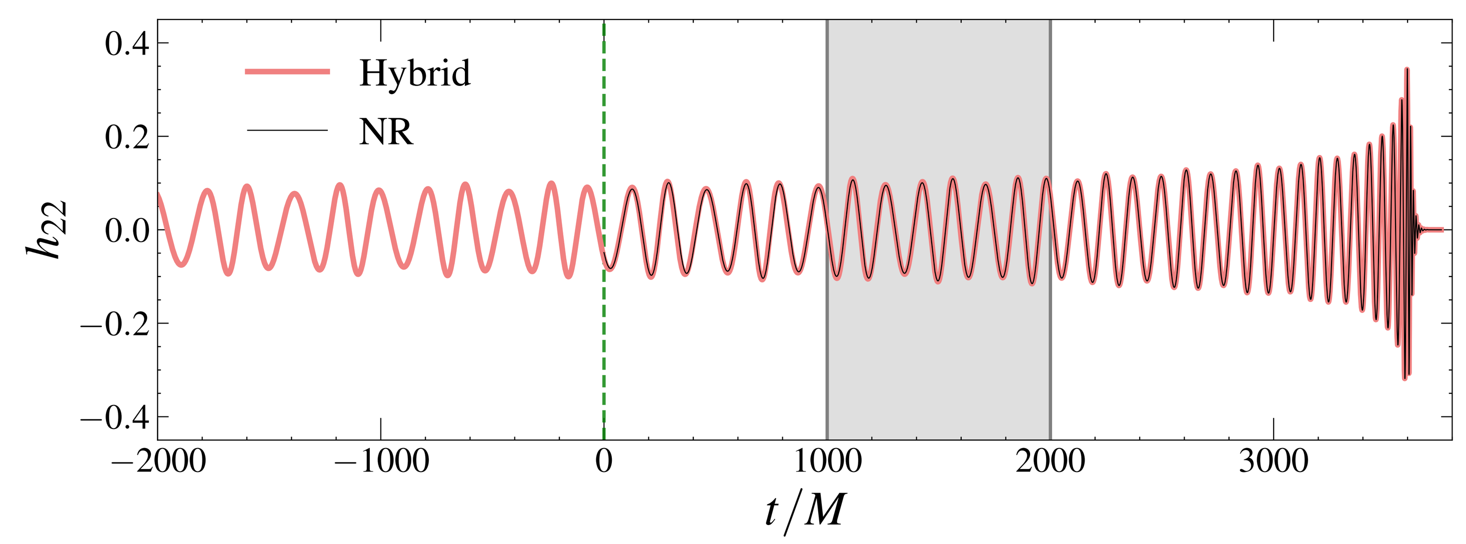

Figure 2. A dominant mode hybrid with a PN model describing the early inspiral, and the late-inspiral, merger, ringdown stages described by the NR simulation.

Overlapping hybrid and NR waveforms outside (on the left of) the matching window hint at the quality of hybridisation performed here.

Effect of eccentricity

Inability of a quasi-circular inspiral-merger-ringdown model (formally coded in **LALSuite**[1] as **IMRPhenomD**) in extracting gravitational wave signals from even mildly eccentric binary black holes is demonstrated through **mismatch** calculations assuming hybrids as targets.

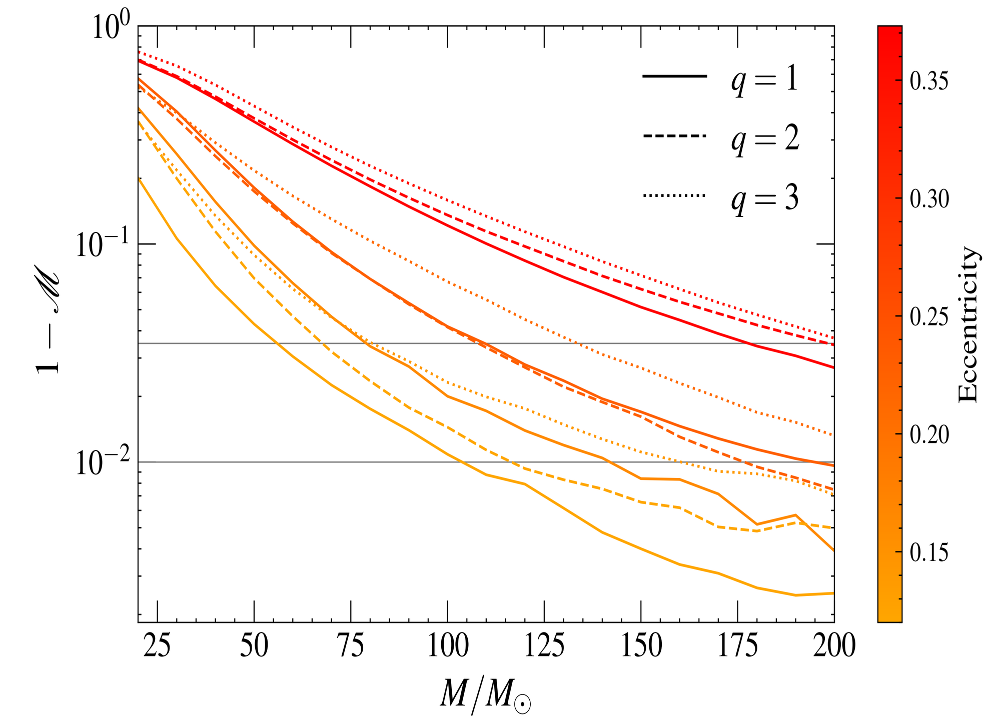

Figure 3. Mismatches between quasi-circular waveforms of IMRPhenomD family and the eccentric hybrids constructed here with initial eccentricities ranging between $0.12 \leq e_0 \leq 0.37$, with varying total mass. The two horizontal lines represent 96.5% and 99% agreement respectively.

A time-domain Model

Amplitude model displayed in Figure 4 is a proof of principle demonstration for constructing a time-domain model by stitching together a post-Newtonian waveform (**EccentricTD**) and a circular model (**EoBNRv2**).

Model highlights

- The amplitude model presented here has a mismatch $< 1\%$ with the target hybrid model.
- Constructed out of waveforms already coded-up in LALSuite[1] that are publicly available.

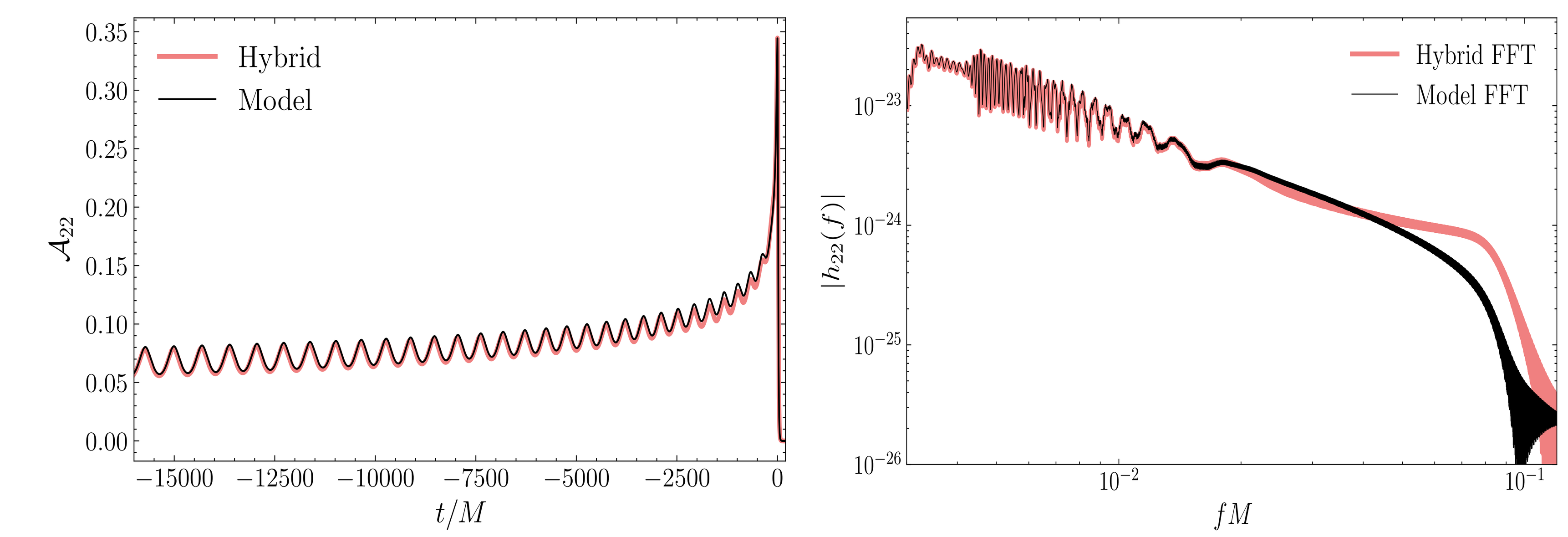

Figure 4. [Left panel] Comparison of a time-domain eccentric model (constructed out of stitching an inspiral and a merger-ringdown model) with a target hybrid. [Right panel] Amplitude of the Fast-Fourier-Transforms of the model and the hybrid.

A frequency domain model

A frequency domain, amplitude model displayed in Figure 5 is a proof of principle demonstration for constructing a model by stitching together a frequency domain post-Newtonian waveform (here FFT of **EccentricTD**) and a circular model (**PhenomD**).

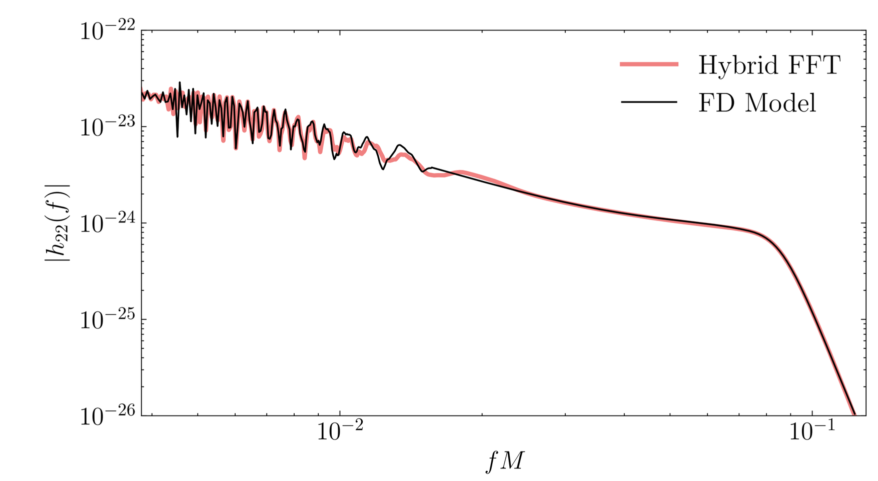

Figure 5. Comparison of a frequency domain eccentric model for signal amplitude with the FFT of the target hybrid.

References

[1] LIGO Scientific Collaboration. LIGO Algorithm Library - LALSuite. free software (GPL), 2018.

[2] Sashwat Tanay, Maria Haney, and Achamveedu Gopakumar. *Physical Review D*, 93(6):064031, 2016.