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Probing Diffuse Dark Matter Haloes with Diffractive Lensing of GW

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Based on “Small-scale shear: peeling off diffuse subhalos with gravitational waves” arXiv : 2103.08618[astro-ph.CO]
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Outline

I. Motivation

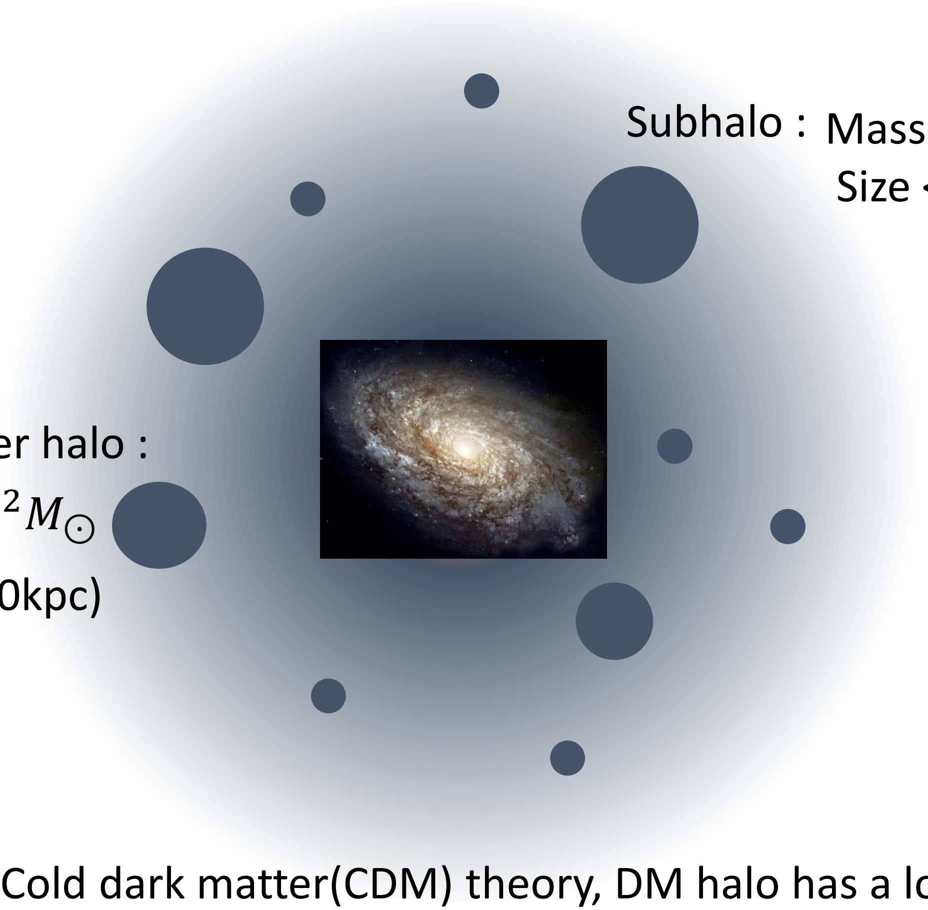
II. Diffractive Gravitational Lensing

III. Detection prospect

IV. Summary

I. Motivation

- Dark matter subhalo



Subhalo : Mass < $10^7 M_\odot$
Size <1kpc

Dark matter halo :
mass~ $10^{12} M_\odot$
size~ O(100kpc)

According to Cold dark matter(CDM) theory, DM halo has a lot of subhalos.

Test of CDM -> How many? How steep?

I. Motivation

Gravitational lensing of **Gravitational Wave Chirps**

: Sensitive to **low mass compact** lens

- Intermediate mass black hole (Lai 2018, Jung 2018)
- Dwarf galaxy (Takahashi 2003, Dai 2018)

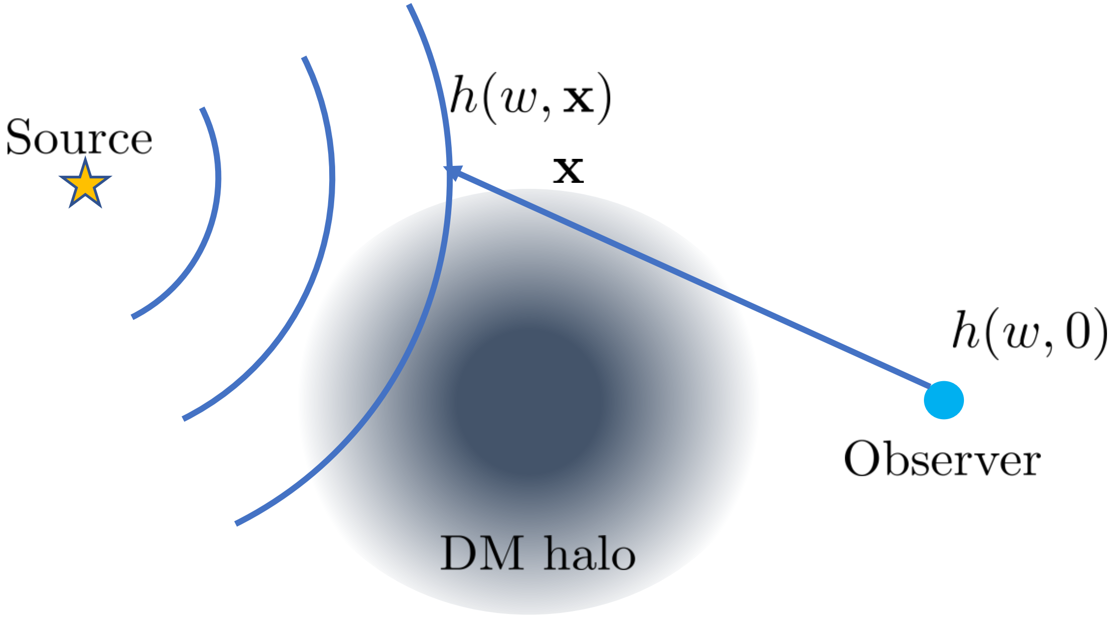
- **Application to Dark matter subhalo ?**
  - Subhalos are diffuse → Weak gravity

  → Only **one signal** perturbed by **Diffraction**

  =Diffractive lensing

We need to understand Diffractive lensing of Gravitational wave.

II. Diffractive lensing : Wave optics



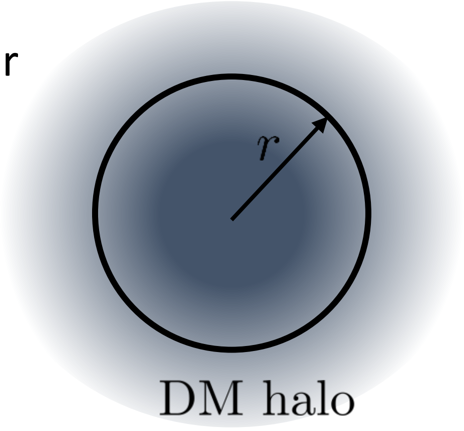
$$(\nabla^2 + w^2)h(w,\mathbf{x}) = 4w^2 U(\mathbf{x})h(w,\mathbf{x})$$ Wave propagation

$$\Rightarrow h(w,0) \simeq \left[\frac{w}{2\pi i d_{\rm eff}}\int d^2x' e^{iwT(x')}\right] h_0(w,0)$$ Kirchhoff integral

$$F(w) \equiv \frac{w}{2\pi i d_{\rm eff}}\int d^2x' e^{iwT(x')}$$ Lensing amplification factor (F=1 for no lens)

$h_0(w,0)$: GW without lensing effects

II. Diffractive lensing : Apprx. Solution



$\bar{\kappa}(r)$: (dimless) **Mean surface density** of DM halo within a radius r

With **weak lensing** assumption, We find that

$$F(w) \simeq 1 + \frac{w}{i d_{\text{eff}}} \int_0^\infty dx\, x e^{i \frac{w x^2}{d_{\text{eff}}}} \,\overline{\kappa}(x)$$

$$\Rightarrow F(w) \simeq 1 + \overline{\kappa}\left(\frac{r_F}{\sqrt{2}} e^{i\pi/4}\right) \qquad r_F \equiv \sqrt{\frac{2 d_{\text{eff}}}{w}}$$

$$d_{\text{eff}} = \frac{d_l (d_s - d_l)}{d_s} \ : \ \text{effective distance between lens and observer}$$

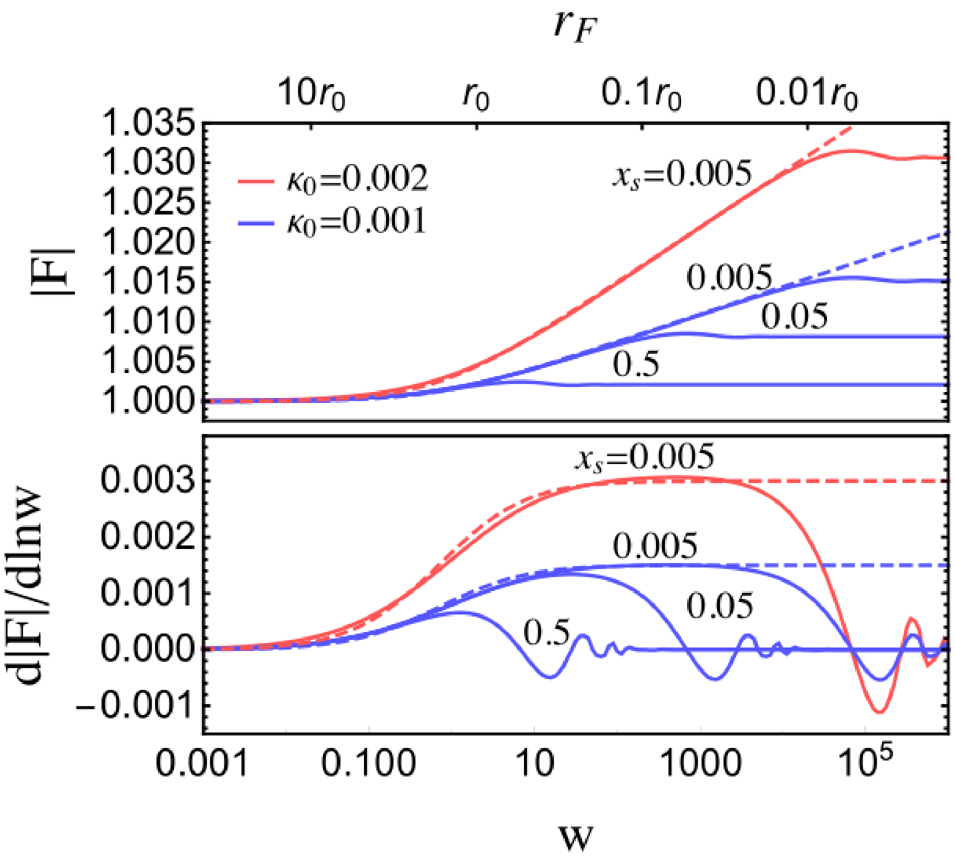
- F(w) is equivalent to **DM Halo profile.**

- We can easily find **F(w) analytically** if mean surface density is given by analytic function.

II. Diffractive lensing : Apprx. Solution

Application to more diffuse DM halo profile : Navarro-Frenk-White(NFW) profile

$$\overline{\kappa}(x) = \frac{6\kappa_0}{x^2}\left[\ln\frac{x}{2} + \mathcal{F}(x)\right] \qquad \mathcal{F}(x) = \begin{cases} \frac{\mathrm{arctanh}\sqrt{1-x^2}}{\sqrt{1-x^2}} & x < 1 \\ 1 & x = 1 \\ \frac{\arctan\sqrt{x^2-1}}{\sqrt{x^2-1}} & x > 1 \end{cases} \qquad x = r/r_0$$



Solid : Numerical integration

Dashed : Analytic solution

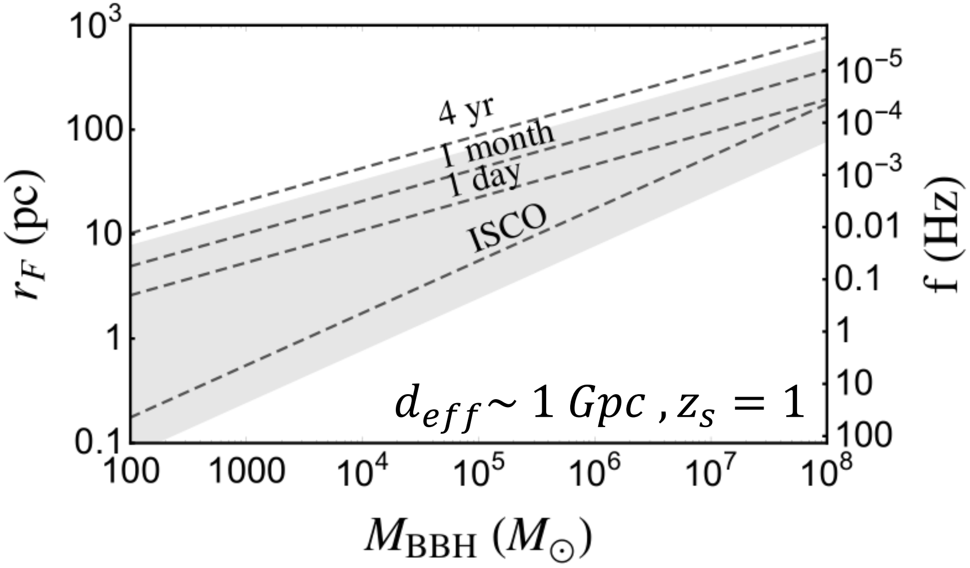
II. Diffractive lensing : Condition

Due to wave optics effects,

Point source has an **effective source size** (Fresnel length), $\boldsymbol{r_F} = \sqrt{\frac{\boldsymbol{2d_{eff}}}{\boldsymbol{w}}}$.

$$r_F \simeq 1.76\mathrm{pc}\sqrt{\left(\frac{d_{\mathrm{eff}}}{\mathrm{Gpc}}\right)\left(\frac{\mathrm{Hz}}{f}\right)} \sim (\text{sub halo length scale})$$

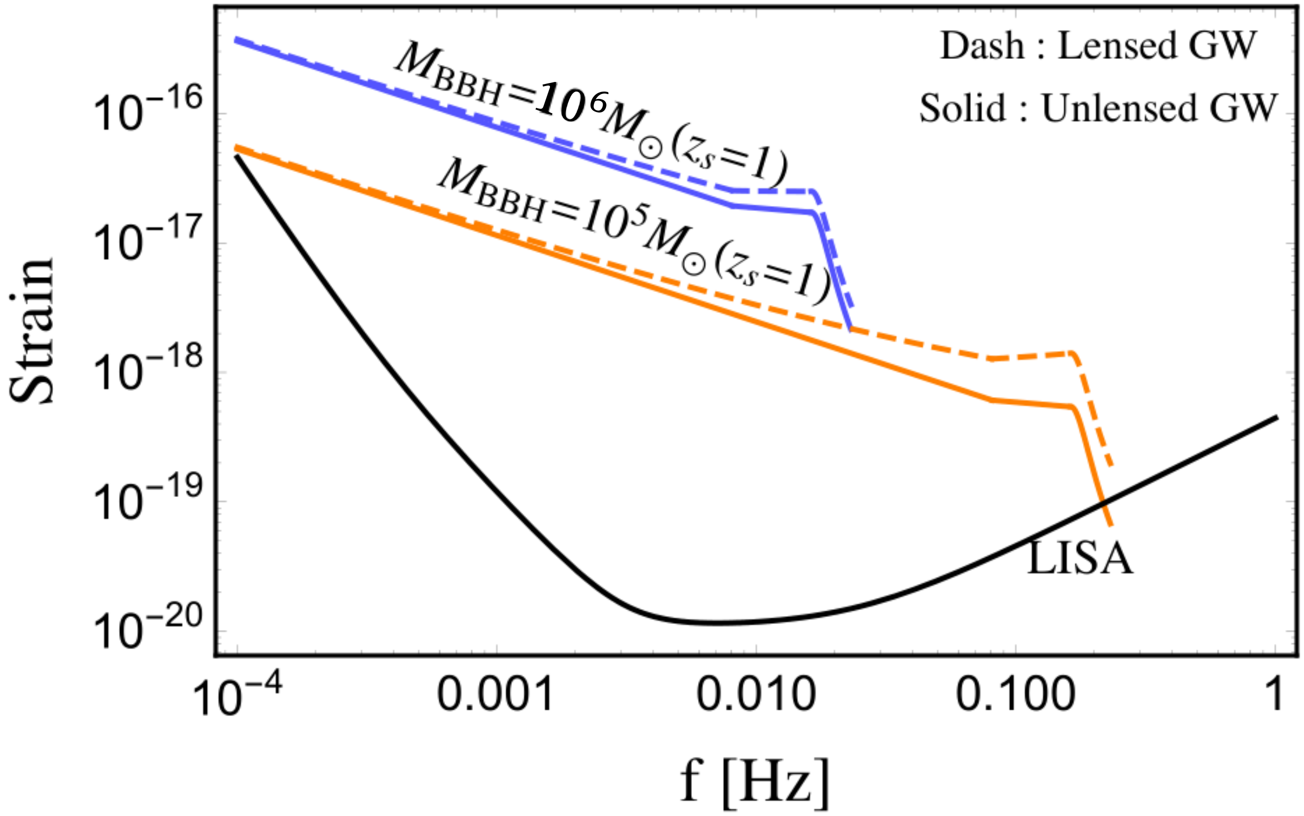
GW chirps from **massive Black hole binaries** :
low frequency, large source distance, broad spectrum



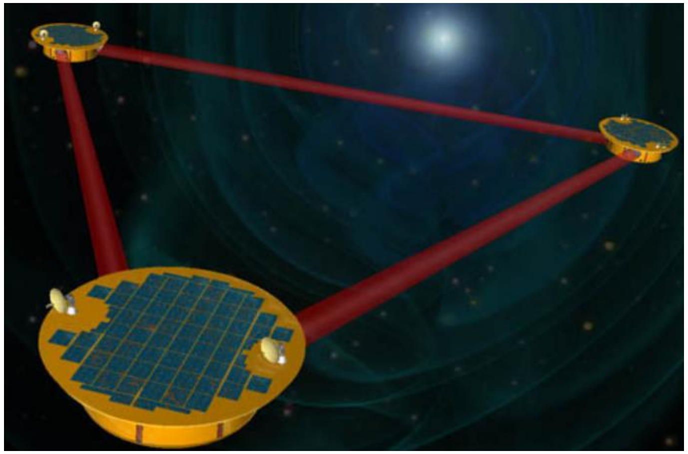
II. Diffractive lensing : GW spectrum

Diffractive lensing-induced chirping GW spectrum

Lensing by $\bar{\kappa}(r) \propto r^{-1}$ lens ($M = 10^5\, M_\odot, z_l = 0.35$)

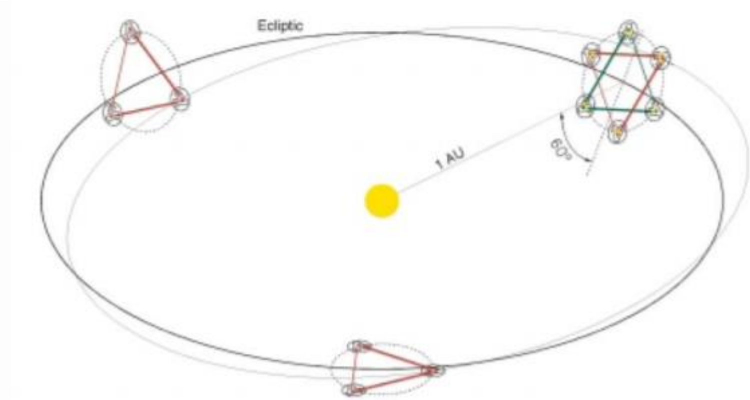


III. Detection prospect : GW detector



Laser Interferometer space antenna(LISA)

Sensitive at 1mHz



Big Bang Observer(BBO)

Sensitive at 0.1 Hz

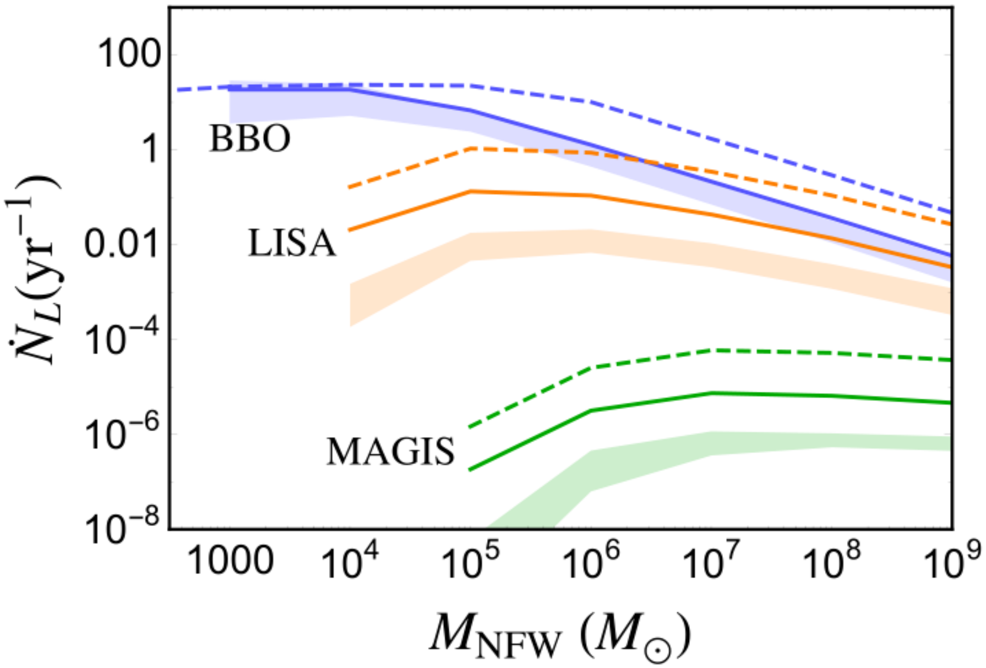
III. Detection prospect

$\dot{N}_L$: Lensing detection per year

- BBO can detect $10^{3-4}M_{\odot}$ halo lensing O(10) per year.

  In future, BBO will discriminate CDM and the other DM models.

- LISA and the others are less promising.
  - Lack of **High Signal-to-Noise Ratio(>1000)** BBH sources



BBH Merger rate
Solid : 0.01 $Gpc^{-3}yr^{-1}$
Shaded : astrophysical
(Bonetti 2018)

IV. Summary

1. Diffraction effects of GW can be significant due to its large **effective source size(Fresnel length $\boldsymbol{r_F}$)**.

2. We can detect and measure DM subhalo profile by diffractive lensing of GW since **F(w) tell us mass distribution at $\boldsymbol{r_F}$ scale.**

3. BBO is most promising GW detectors for low mass halo search, which yields O(10) lensing rate for $10^{3-4}M_{\odot}$ haloes.