

# SEARCHING FOR GRAVITATIONAL WAVES THROUGH AN AUTOREGRESSIVE APPROACH

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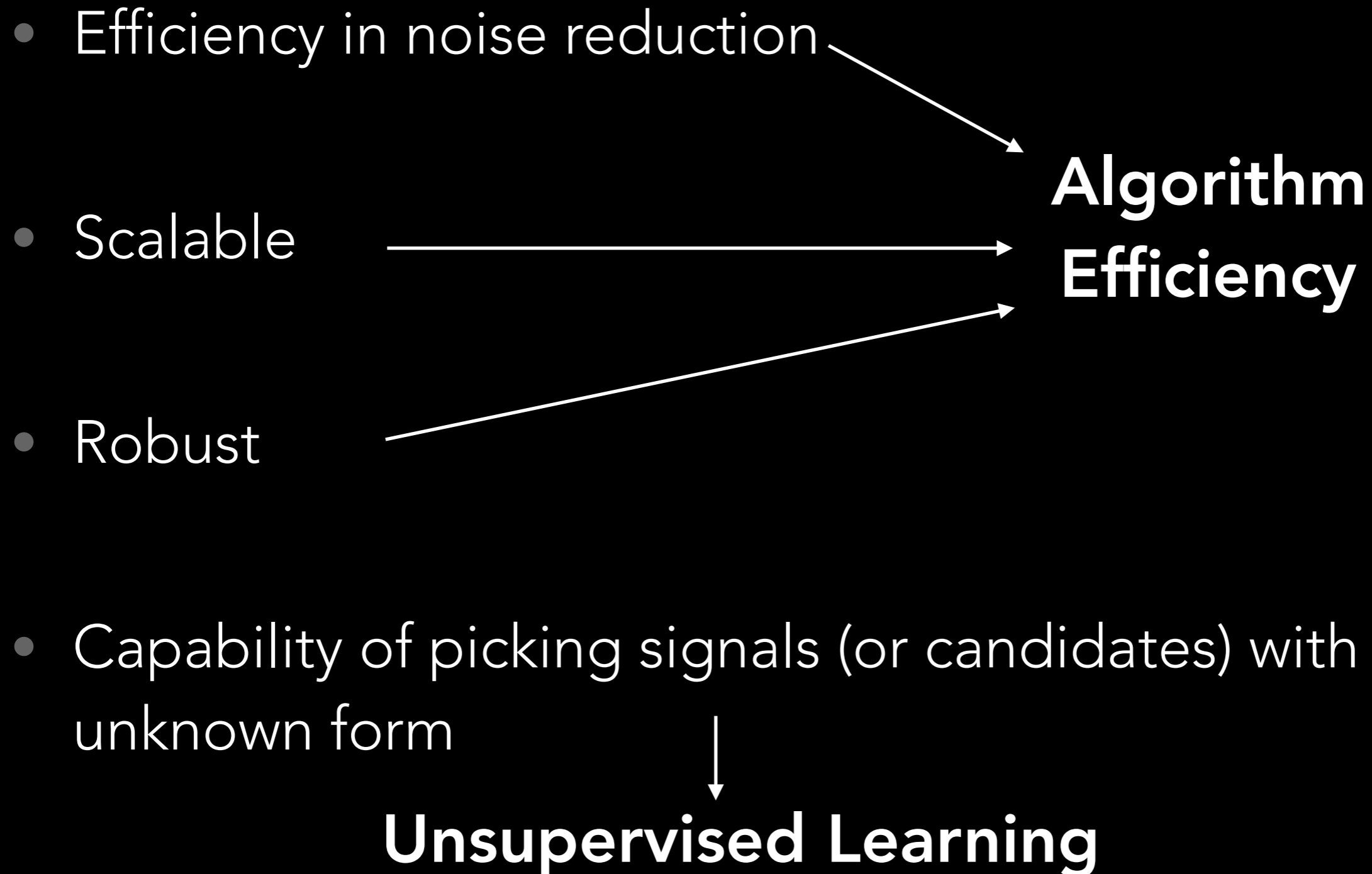
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# COLLABORATORS

- Sangin Kim (CNU, Korea)
- Kwangmin Oh (CNU, Korea)
- Albert Kong (NTHU, Taiwan)
- Lupin Lin (UNIST, Korea)
- Ray Kwan Lok Li (NCKU, Taiwan)
- Alex Leung (MUST, Macau)
- Jianqi Yan (MUST, Macau)
- Shengda Luo (MUST, Macau)
- Hisaaki Shinkai (OIT, Japan)

# WISH LIST FOR DETECTION PIPELINE



# NOISE REDUCTION WITH STOCHASTIC AUTOREGRESSIVE MODELING

- Computationally simple
- Capable to handle various kinds of noise from non-stationary autocorrelated stochastic processes.
- Many applications in diverse fields (e.g. ECG, econometrics), but not many in astronomy until recently (e.g. exoplanet search)

# AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (**ARIMA**) MODEL

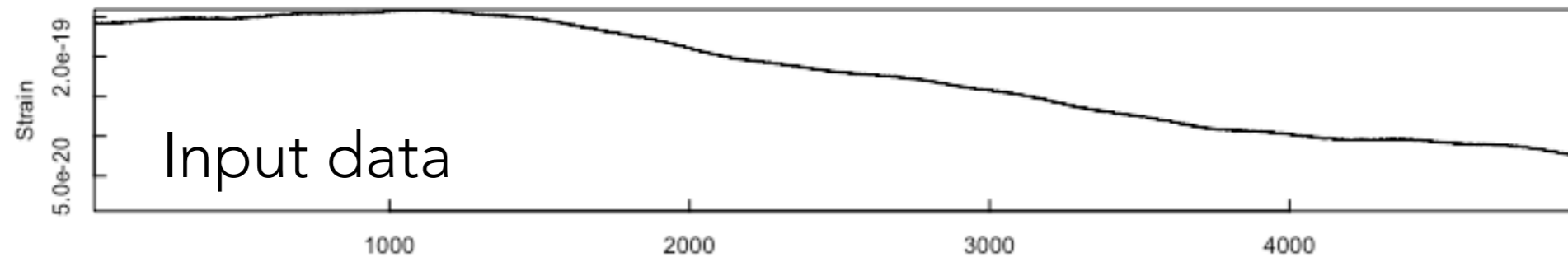
Combining the autoregressive (AR), moving average (MA) and integrated (I) processes together into a single regression procedure, we have ARIMA( $p, q, d$ ) model:

$$(1 - B)^d x_t = \sum_{i=1}^p a_i x_{t-i} + \sum_{j=1}^q b_j \epsilon_{t-j} + \epsilon_t + c$$

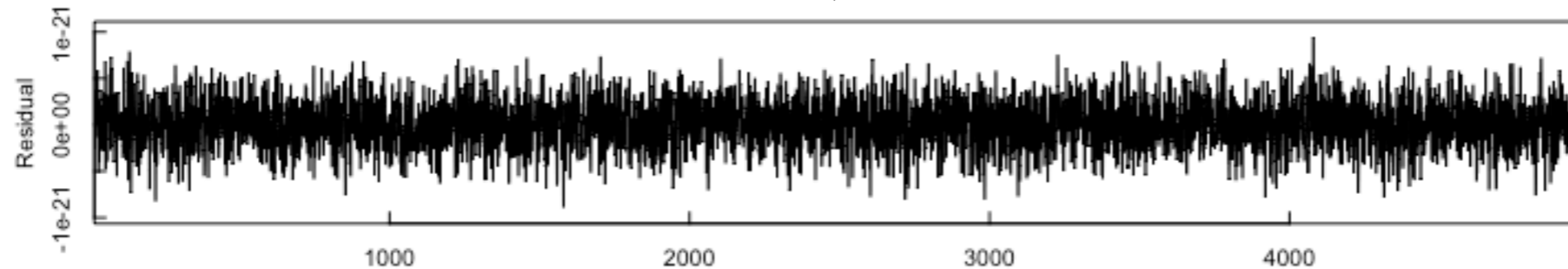
The model parameters can be determined by maximum likelihood estimation with the orders  $p$  and  $q$  determined through certain information criterion (e.g. BIC, AIC).

# PROOF-OF-CONCEPT

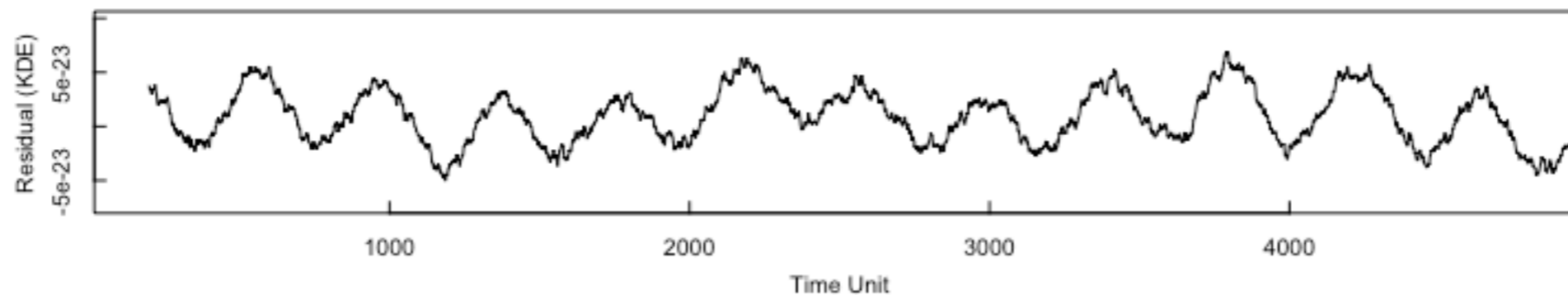
- The simulated LIGO strain series with a constant 10 Hz sinusoidal signal of  $h \sim 10^{-21}$  injected.



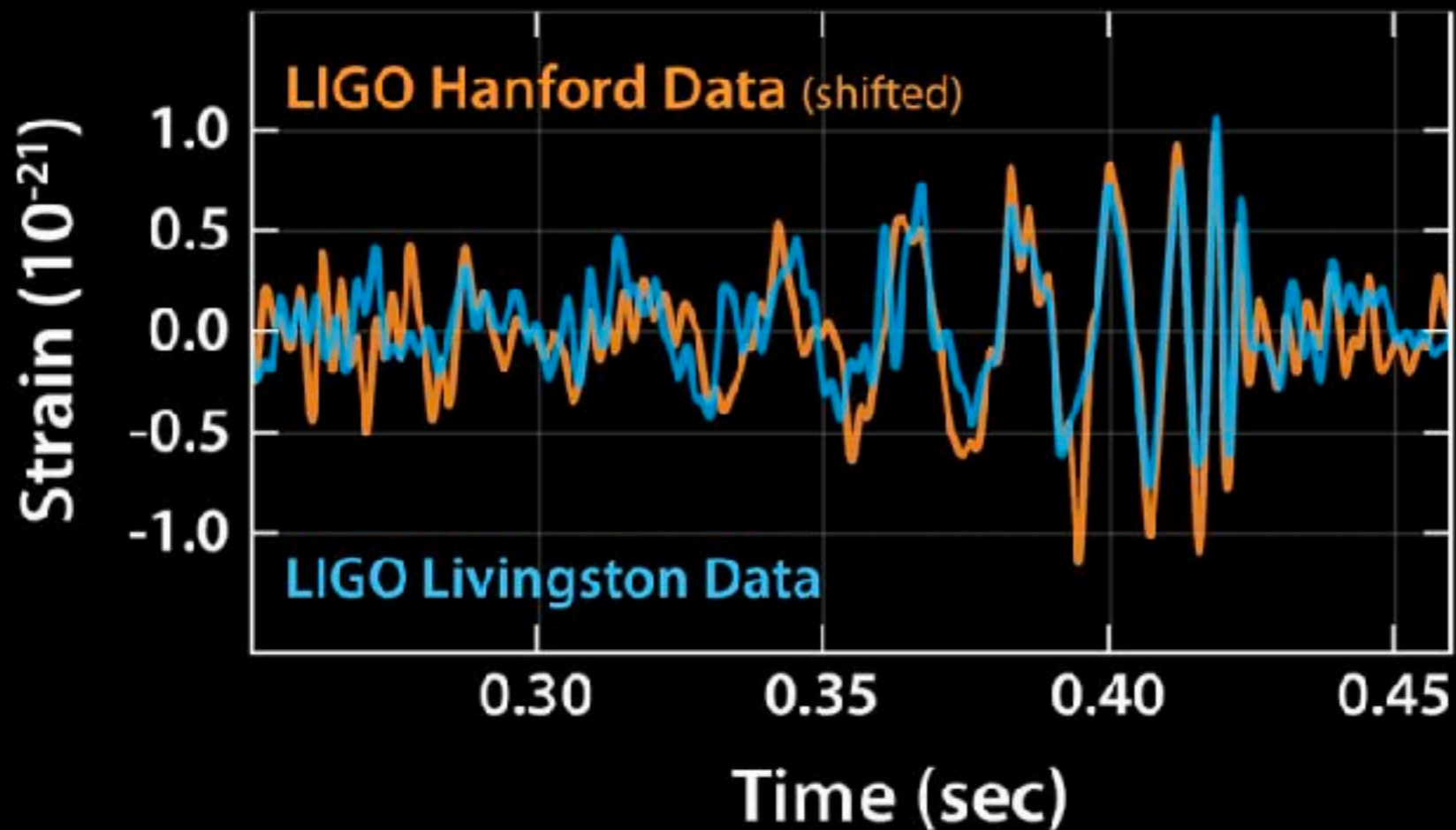
↓ Subtract AR(45)



↓ KDE Low-pass filter

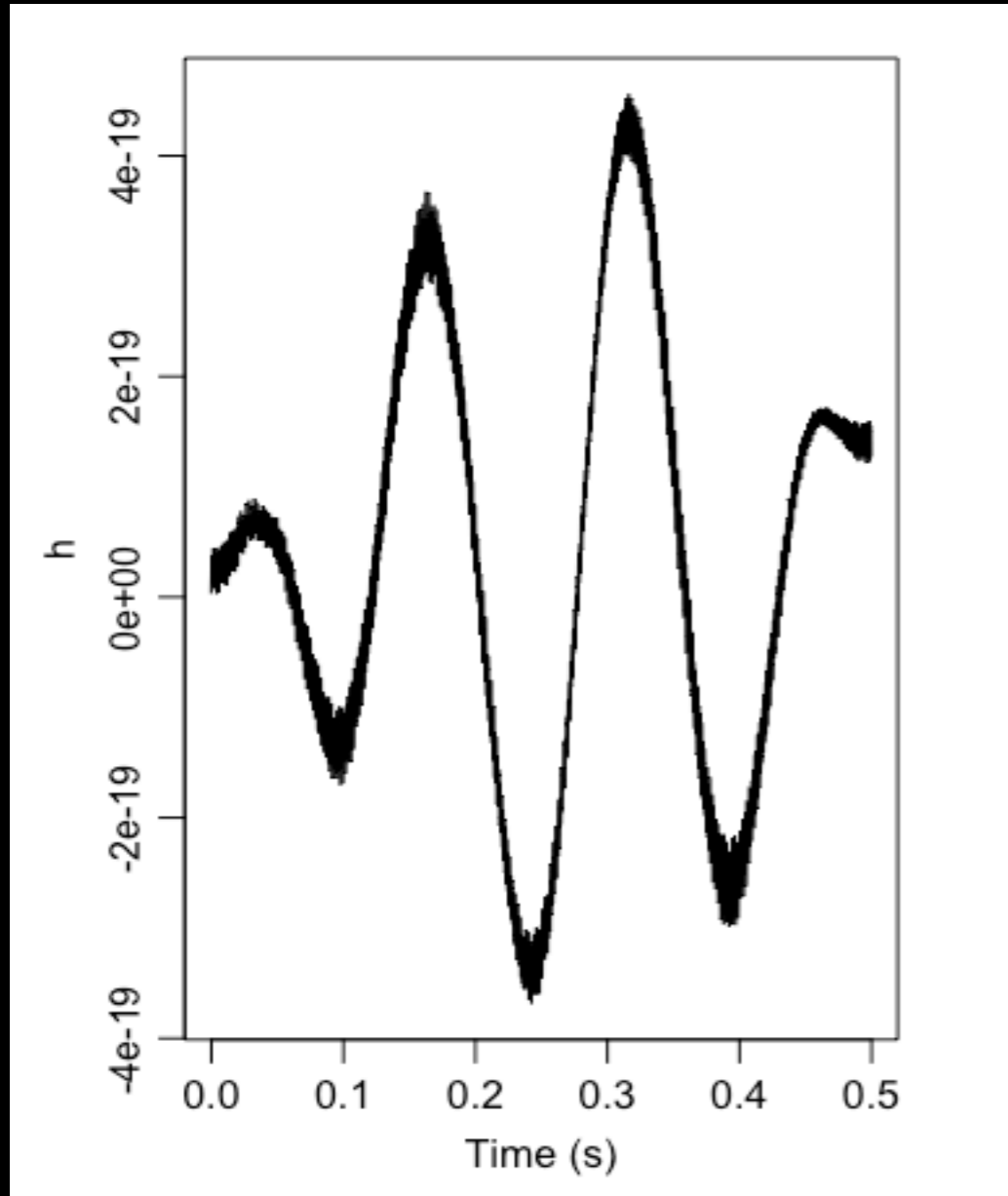


# LIGO DATA OF GW150914



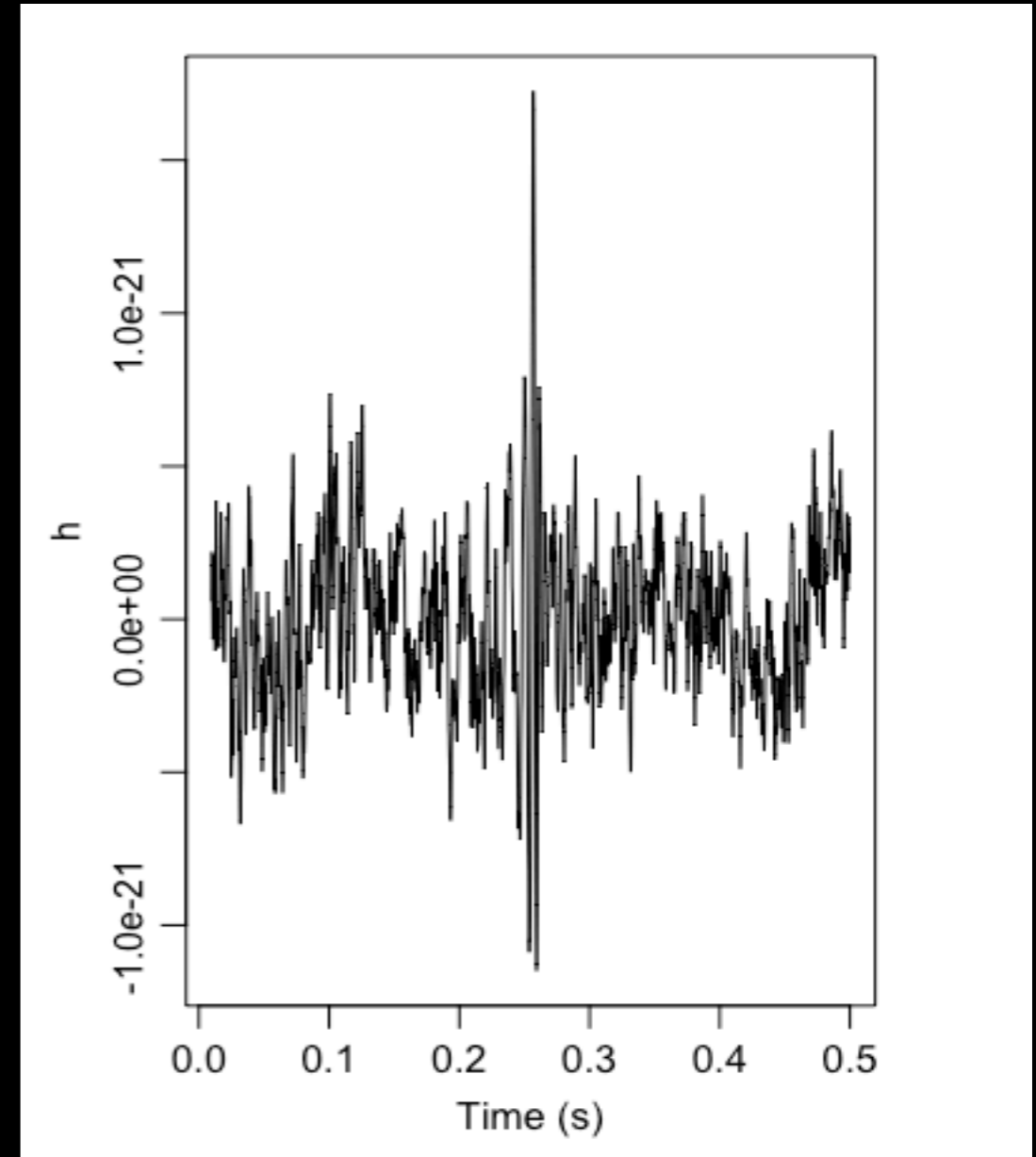
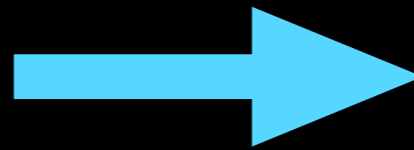
# QUICK LOOK OF GW150914

## I. LIGO Hanford



Raw data

Subtract  
AR(31)  
+  
Low-Pass  
filter

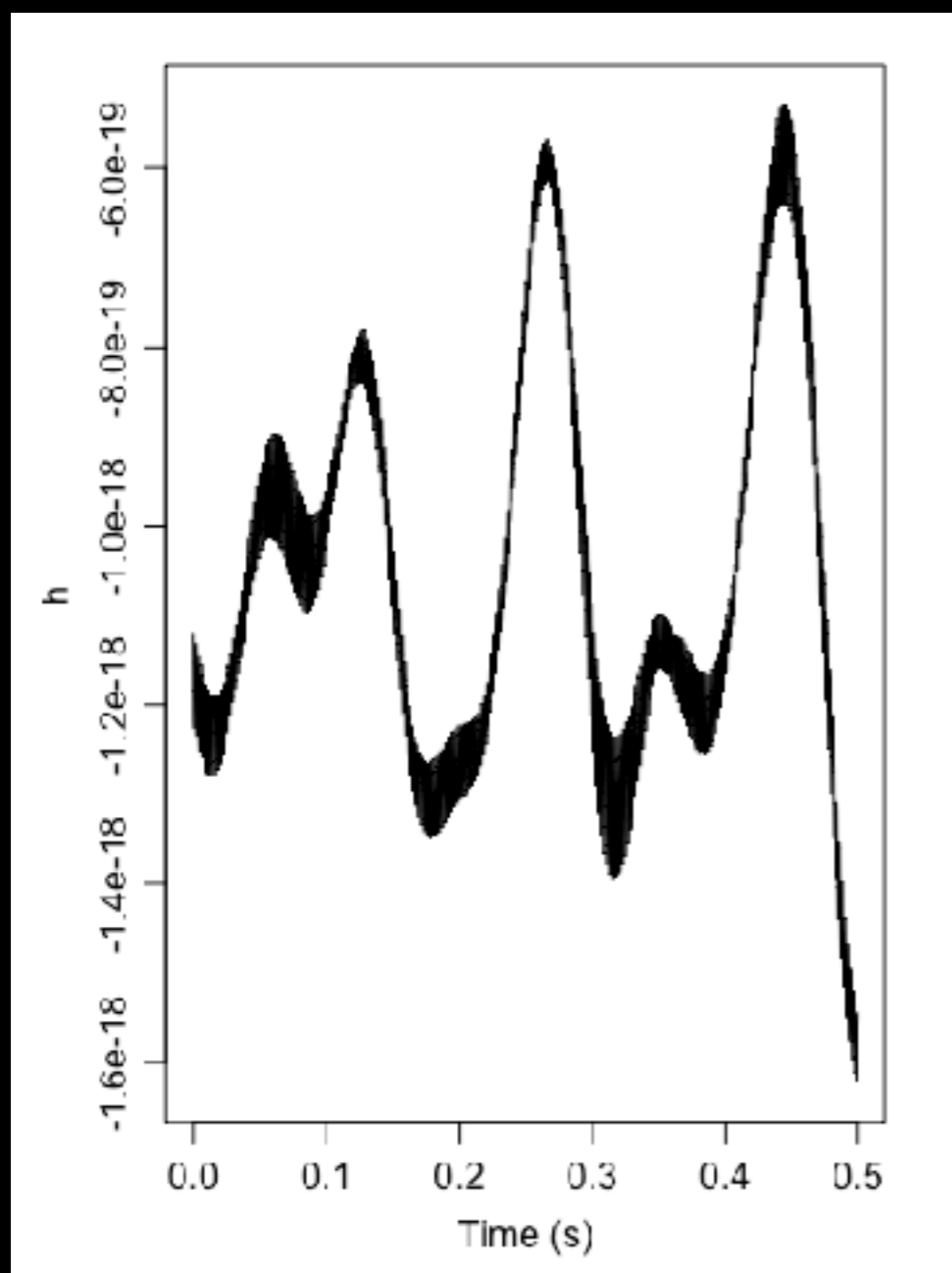


Residuals



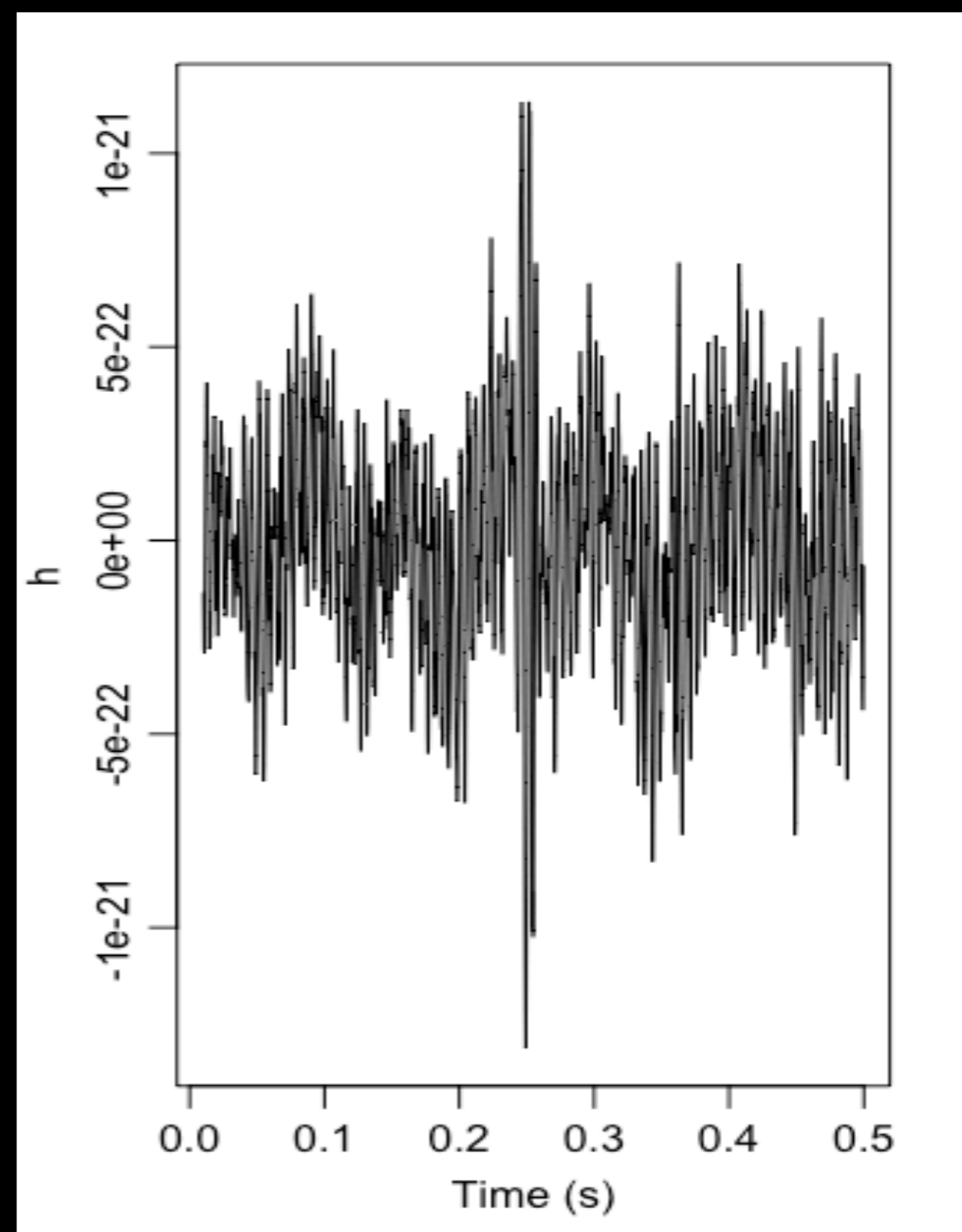
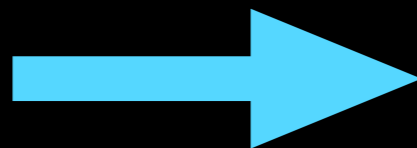
# QUICK LOOK OF GW150914

## II. LIGO Livingston



Raw data

Subtract  
AR(33)  
+  
Low-Pass  
filter



Residuals

# EFFECTIVENESS OF AR MODEL

- A diversity of noises (e.g. glitches) are subsumed into AR model without any fine-tuning and a priori knowledge of the noise nature
- A desirable feature of this method is that the transient GW signals were NOT absorbed by AR model.
- AR is a maximum likelihood estimation procedure which weight all data points equally. As the transient signal is a small fraction in a given window, their data points are essentially ignored in the model.

# OPTIMIZING S/N RATIO

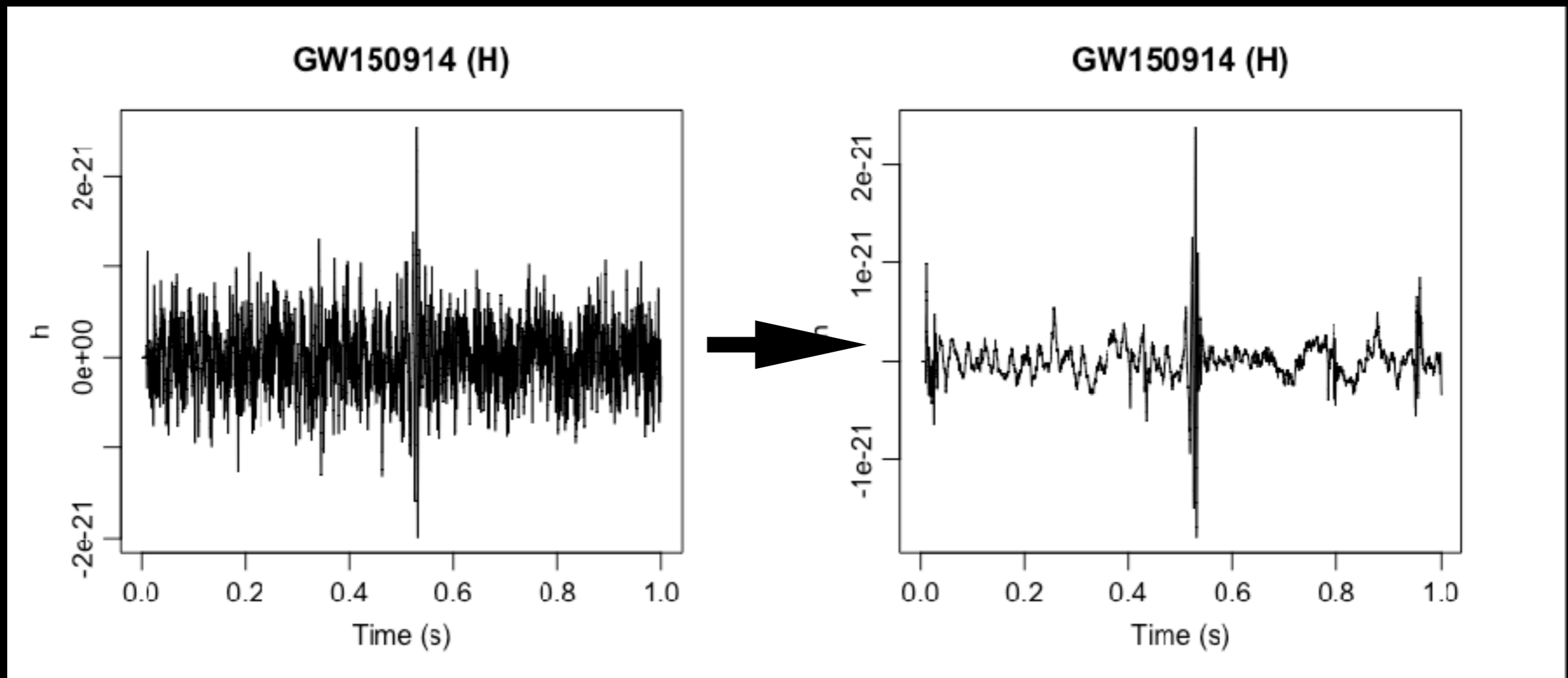
- Kernel size of KDE
- Range of width in an adaptive filter (max./min. Widths)
- Window size

We search for a combination of these hyper-parameters which results in the optimal peak S/N ratio.



# ADAPTIVE FILTERING

- Low frequency noise in the residuals can be further removed by using **repeated median regression** (Siegel 1982)
- Low frequency variation is estimated by median of a sliding window with adaptive size (Gather & Fried 2004).



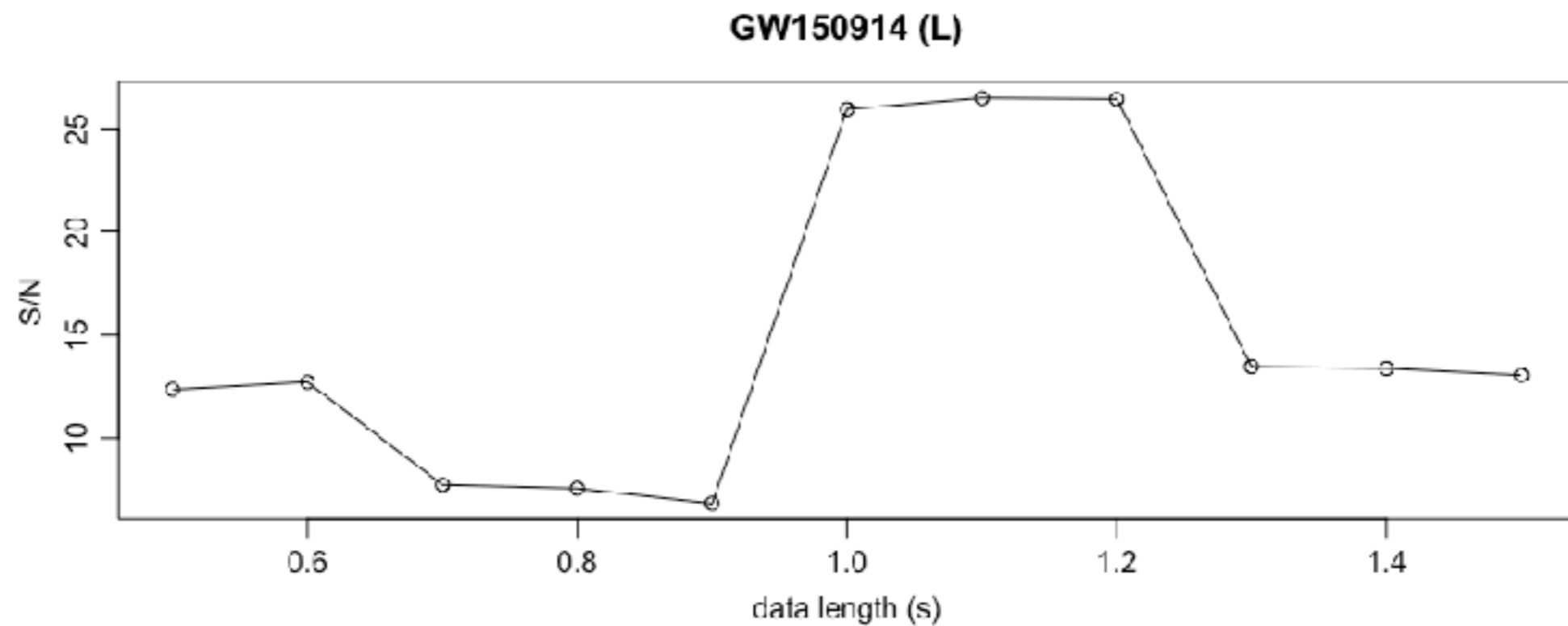
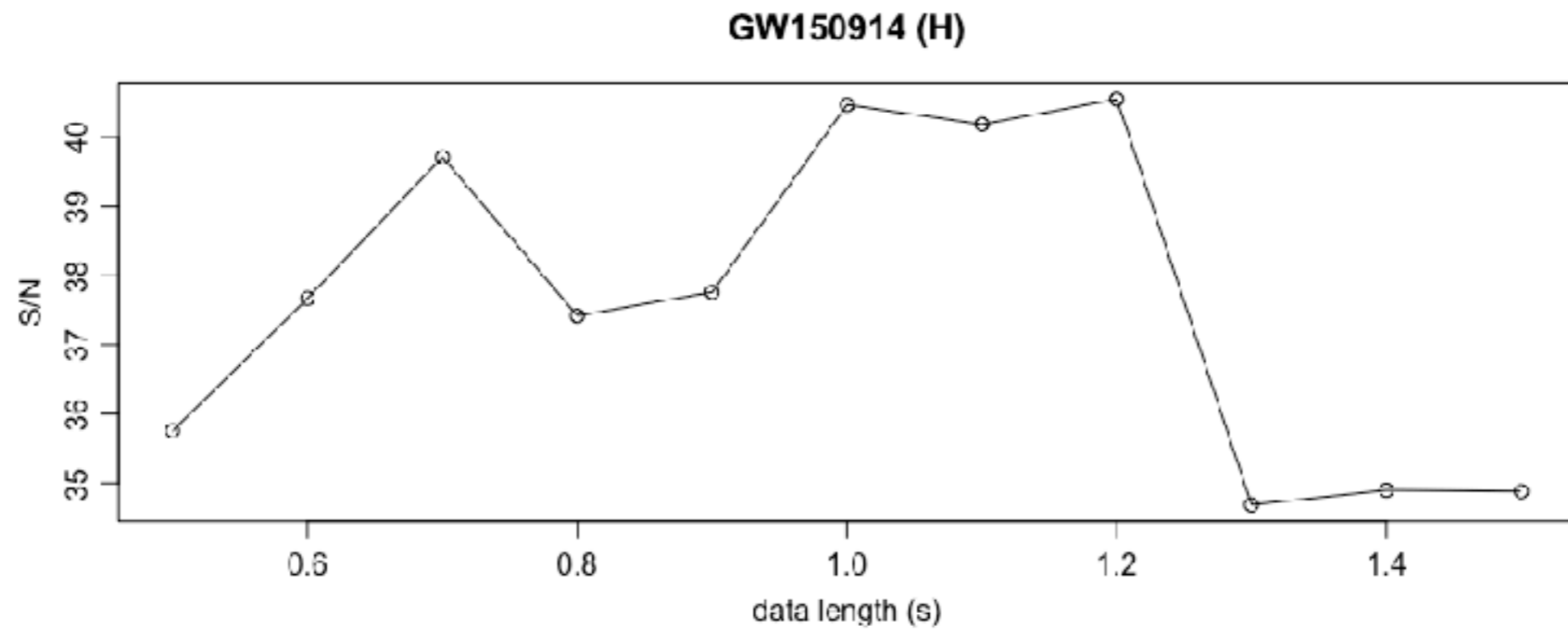
# EFFECTS OF WINDOW SIZE

- Depend on the nature of the event one intend to search (BH-BH meger, NS-NS merger)
- Large window results in small variance and smooth model (*May under estimate the noise*)
- Small window results in small bias and adapts quick to changes. (*May absorb the potential signal*)



**There should be an optimal window size.**

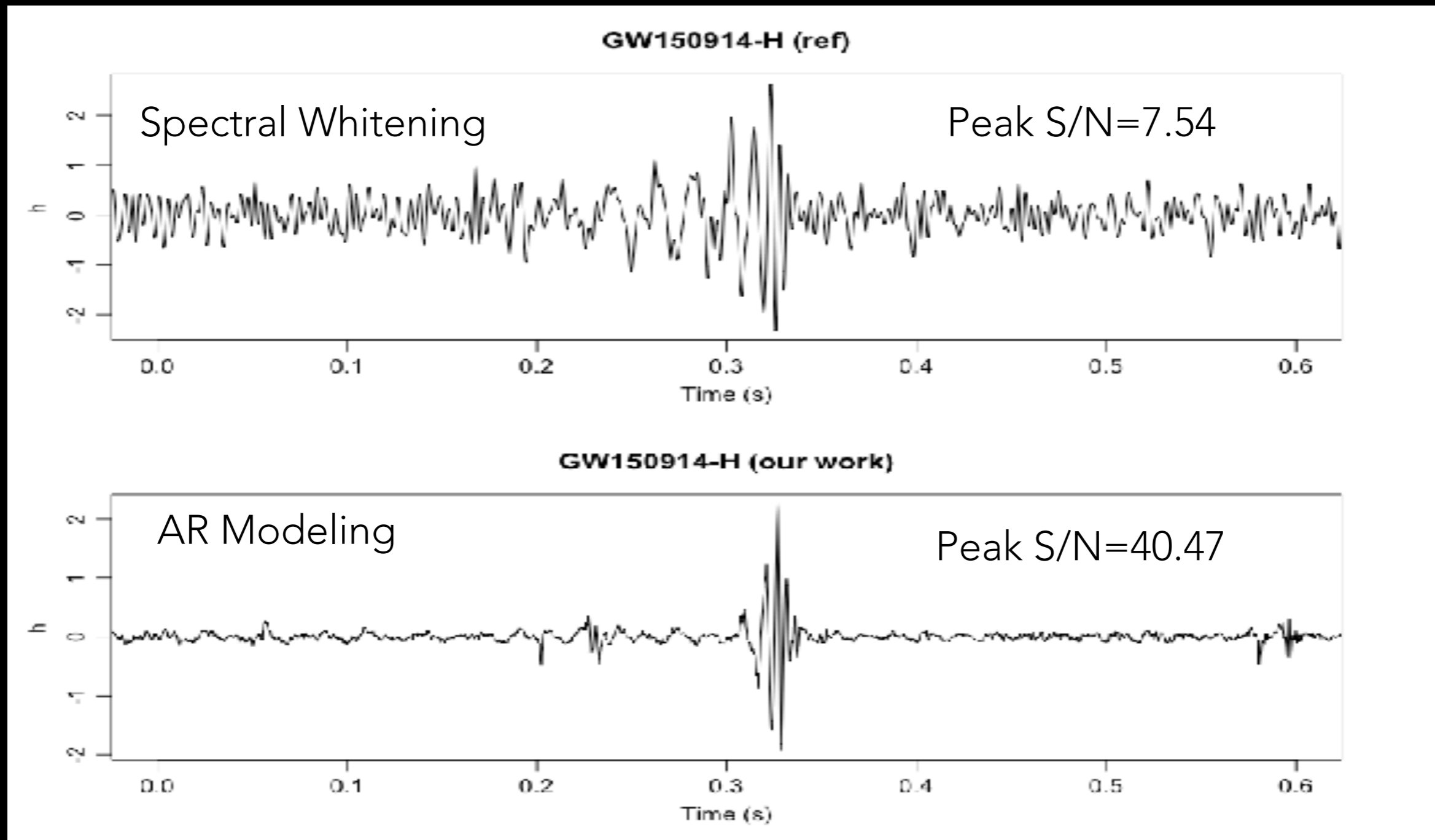
# EFFECTS OF WINDOW SIZE





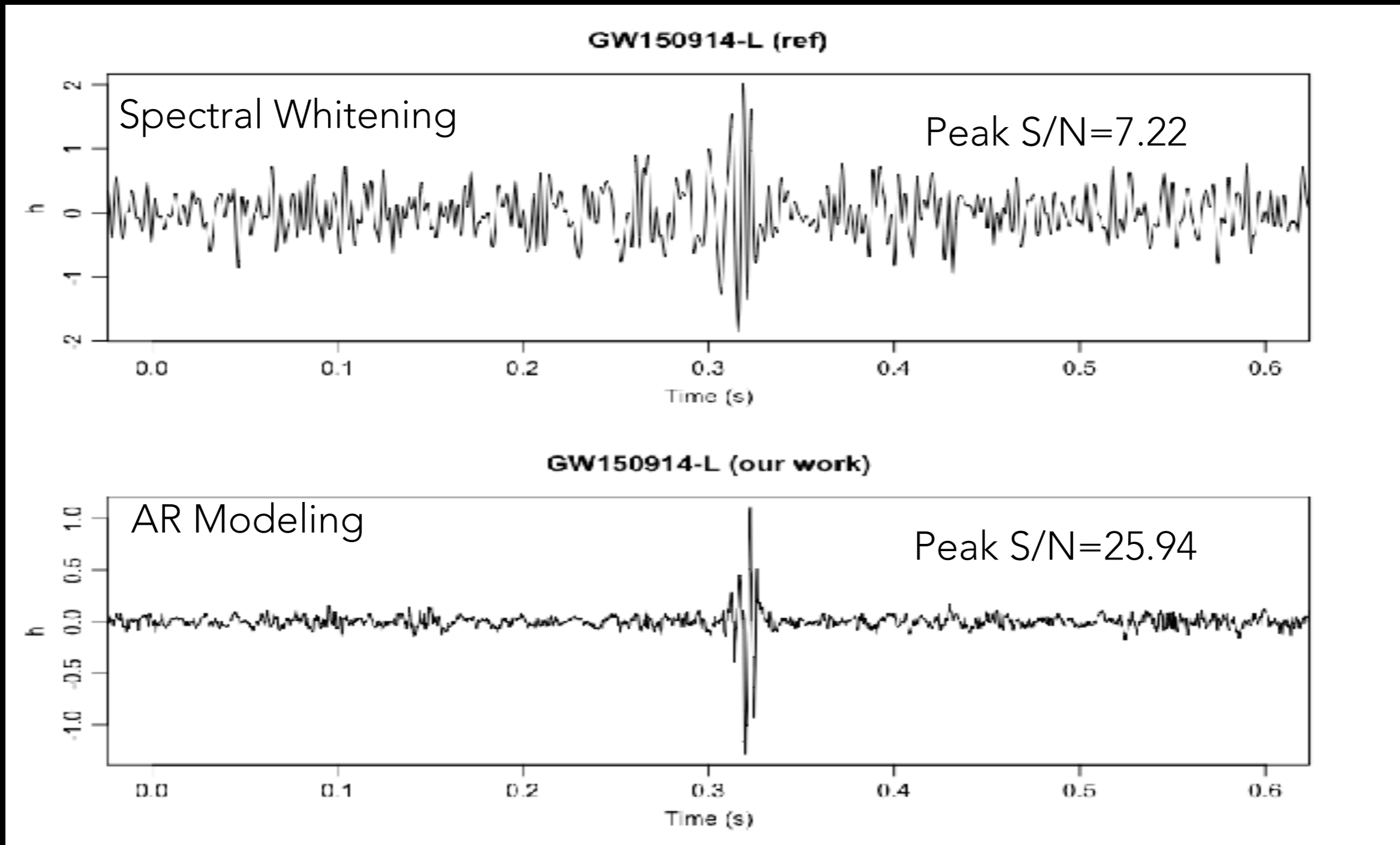
# COMPARISON WITH SPECTRAL WHITENING

- Our framework is capable to attain a better S/N in comparison with the conventional spectral whitening

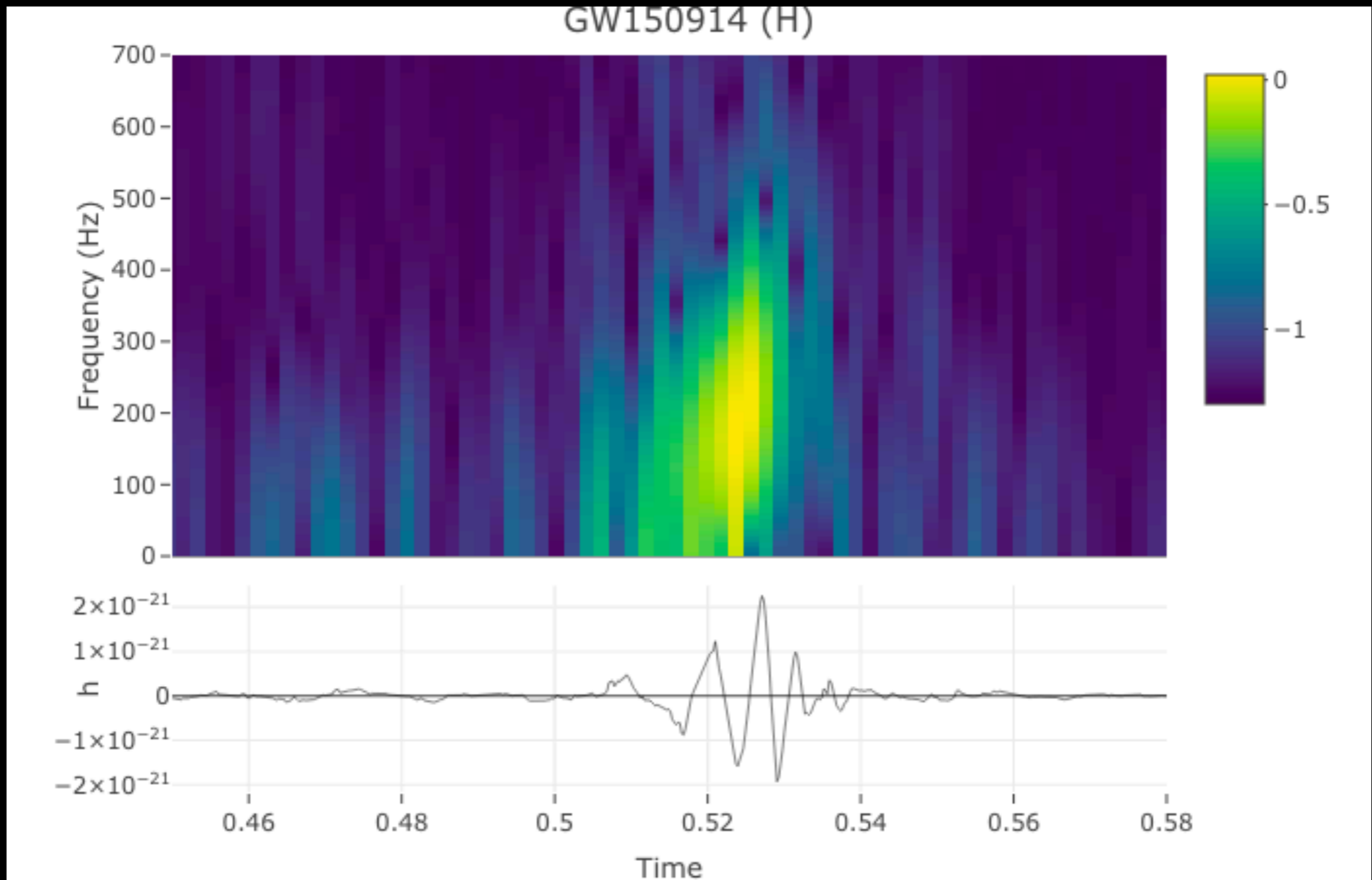


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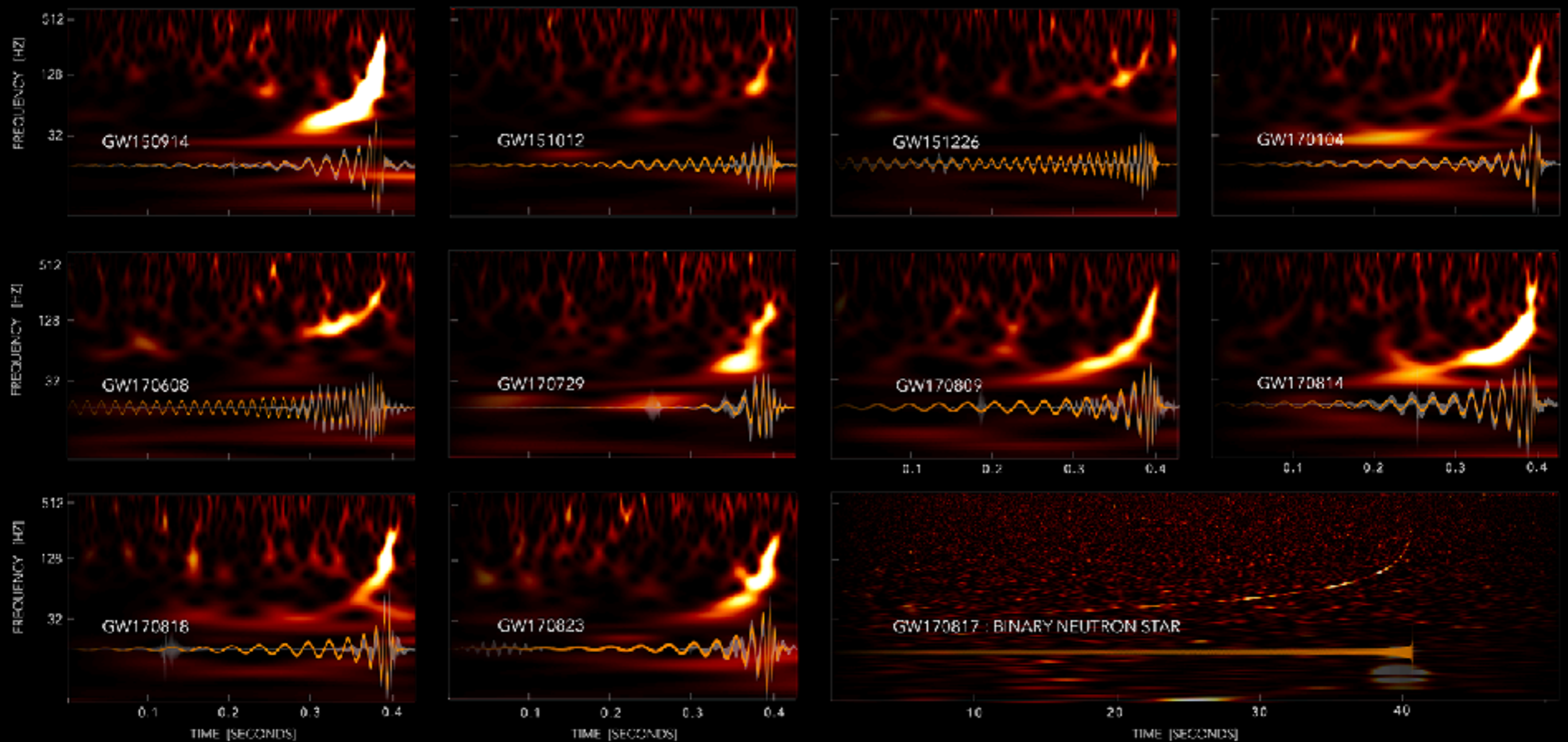
# SPECTROGRAM OF OPTIMAL RESULT





# ON-GOING/FUTURE WORKS

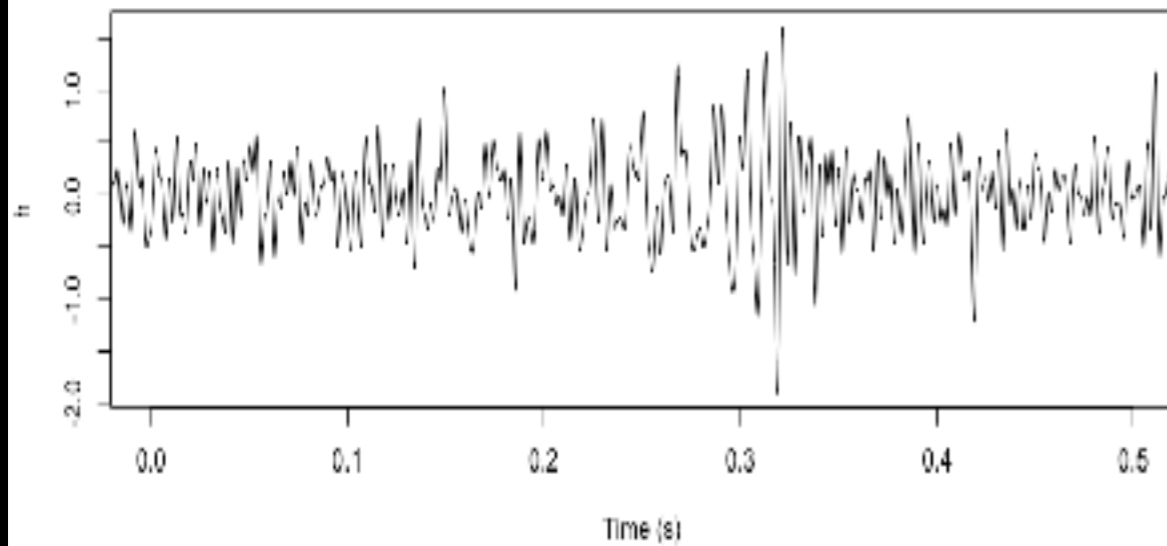
## 1. Recover GWTC-1 events with AR modeling.



# ON-GOING/FUTURE WORKS

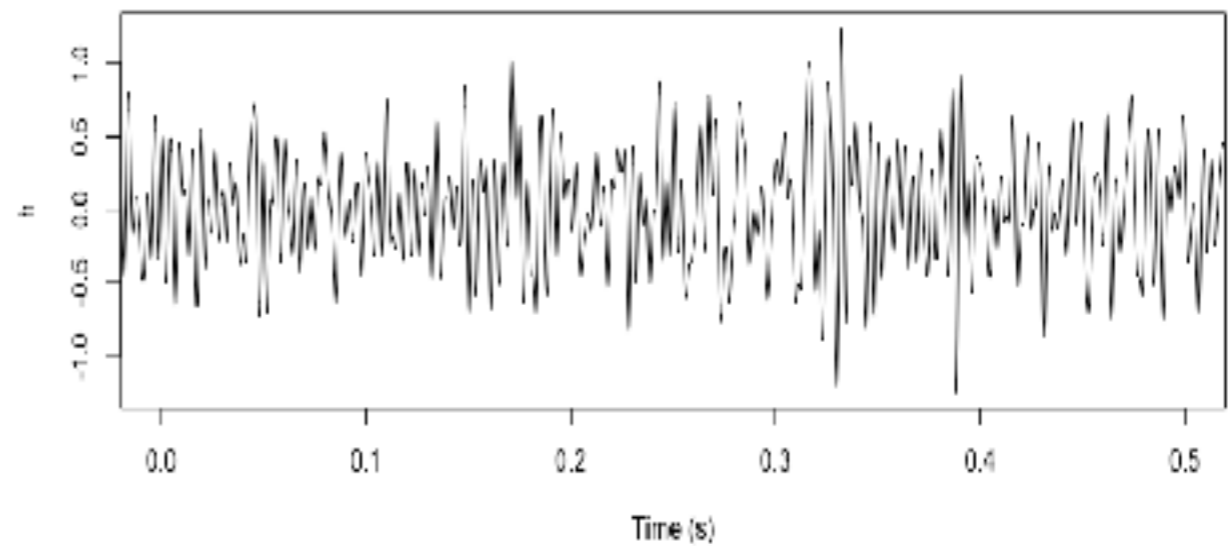
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GW170814-L (ref)

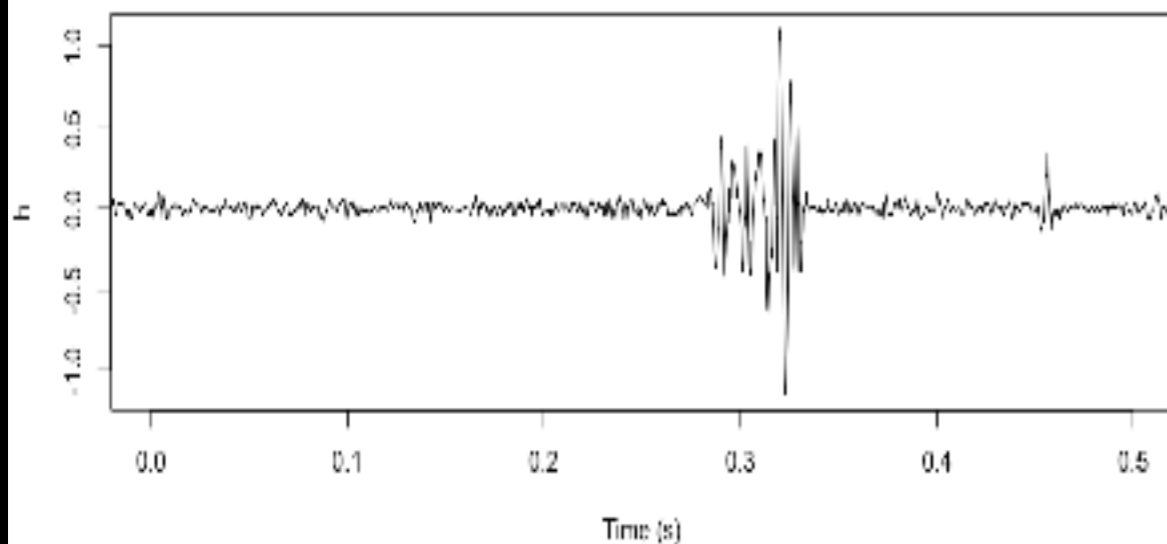


### Spectral Whitening

GW170814-H (ref)

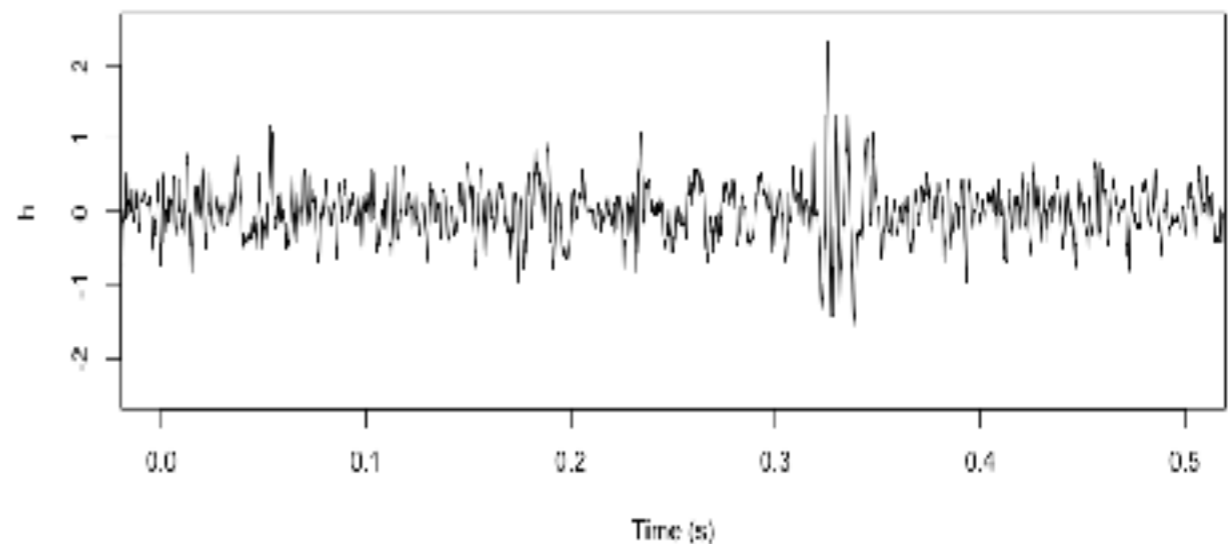


GW170814-L (our work)



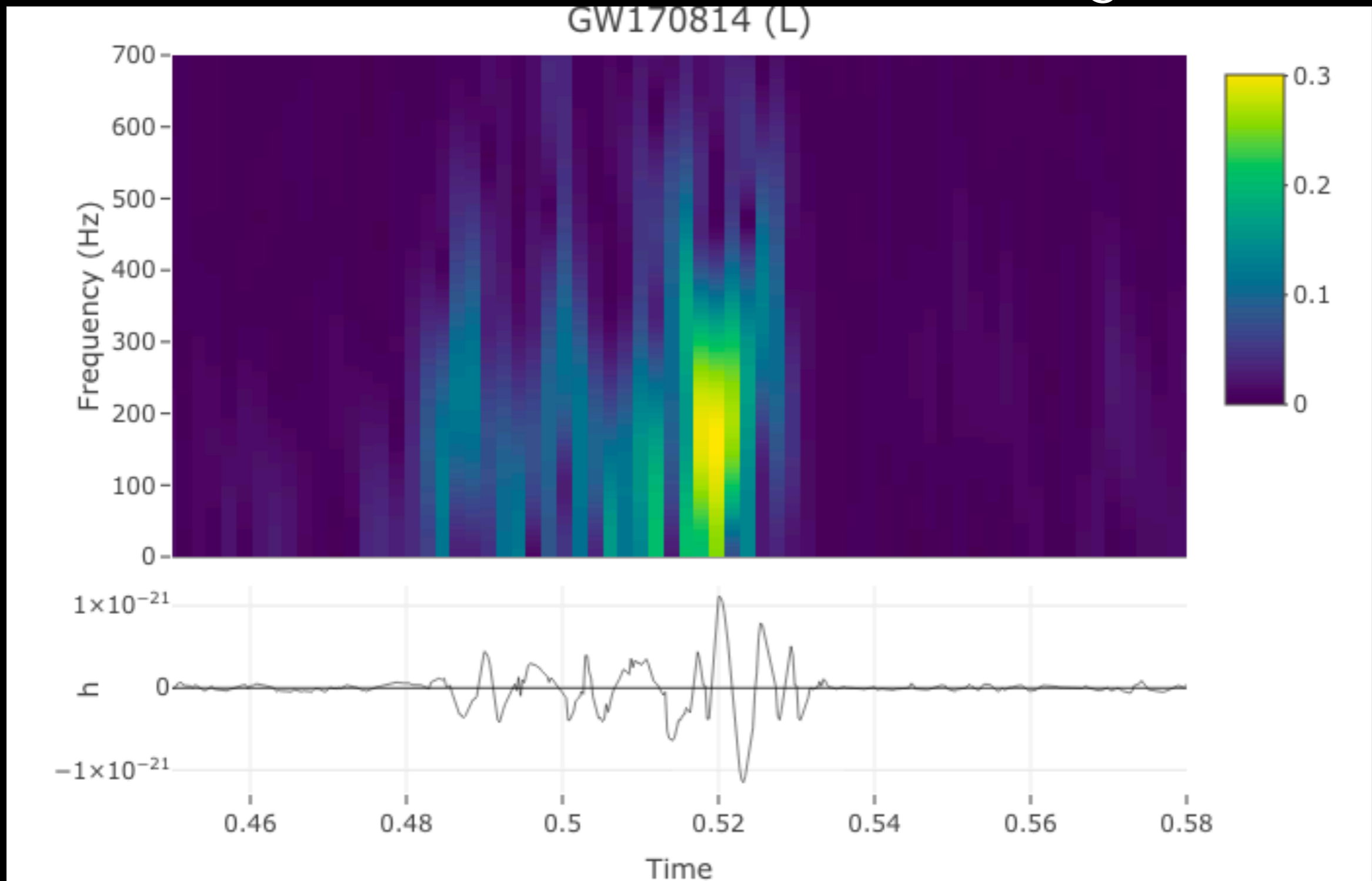
### AR Modeling

GW170814-H (our work)



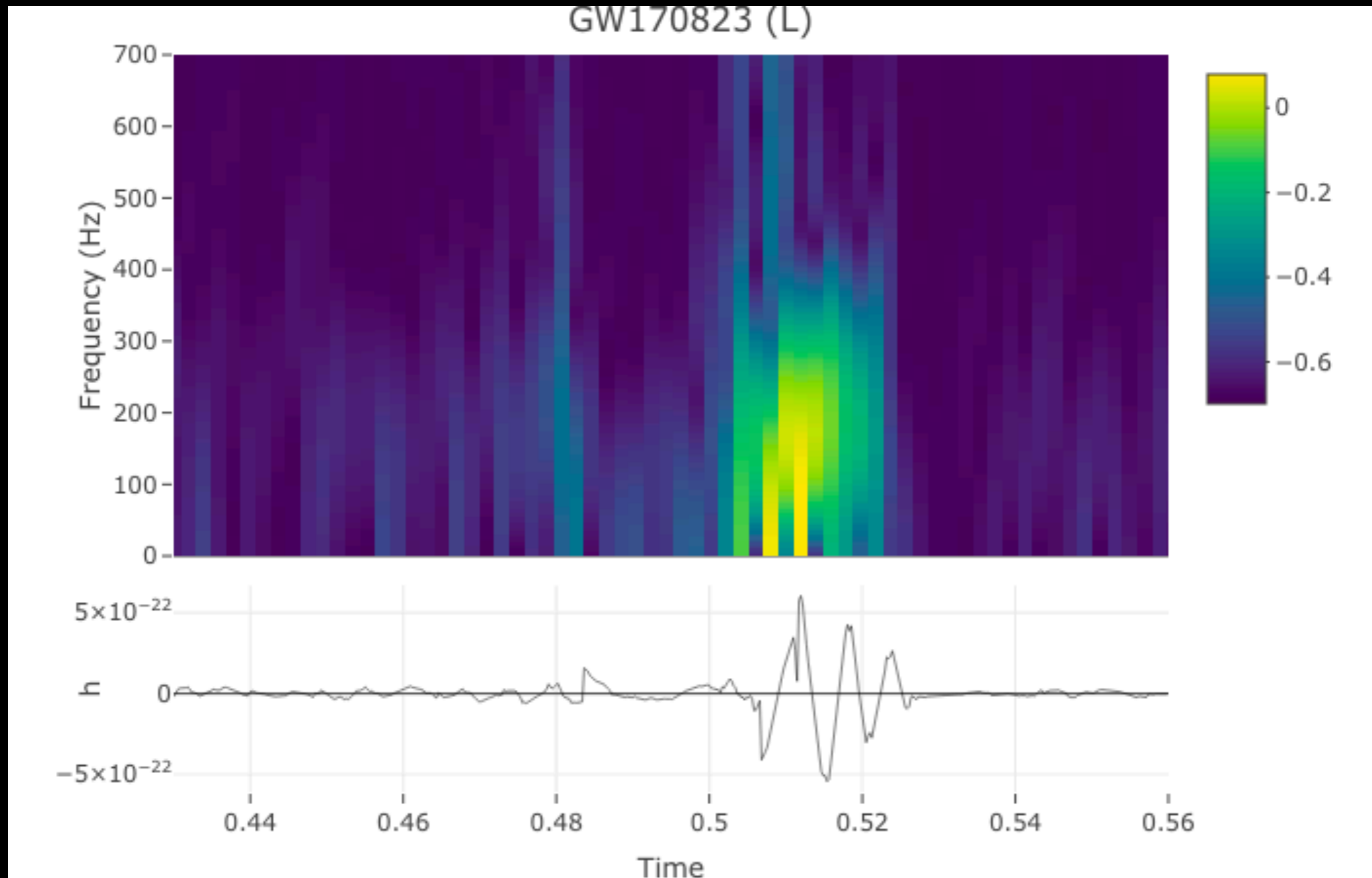
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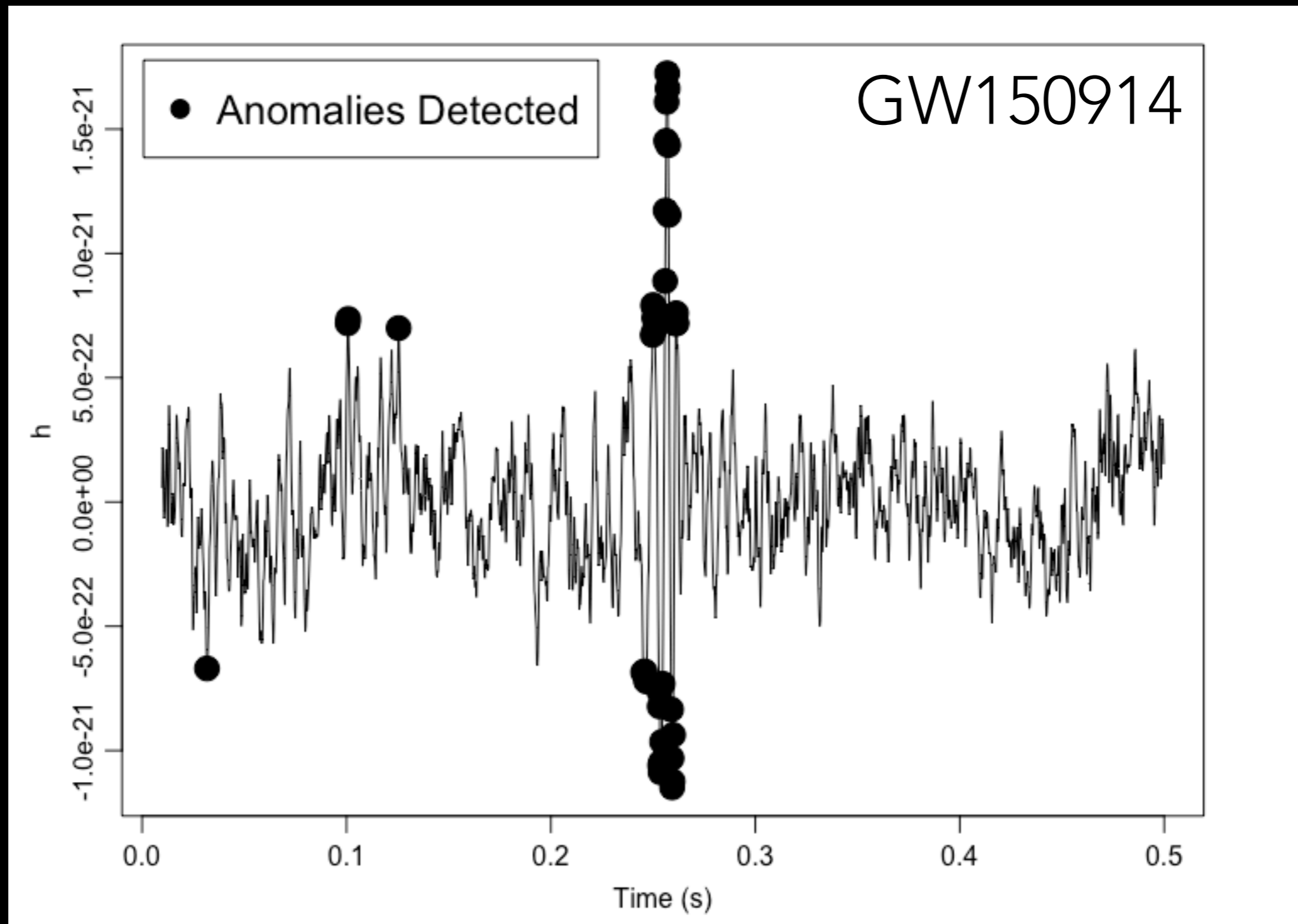
# ON-GOING/FUTURE WORKS

## 2. Anomaly Detections

- After the noise subtraction, events candidates can be identified as anomalies, which differ from normal instances significantly.
- If the duration of the signal is significantly longer than the sampling interval, a cluster of anomalies is expected.
- Anomalies detected from different detectors (LIGO-H, LIGO-L, KAGRA, VIRGO) can be cross-correlated and analysed with clustering technique.
- The shortlisted anomalies can be taken as event candidates for further analysis.

# ON-GOING/FUTURE WORKS

## 2. Anomaly Detections

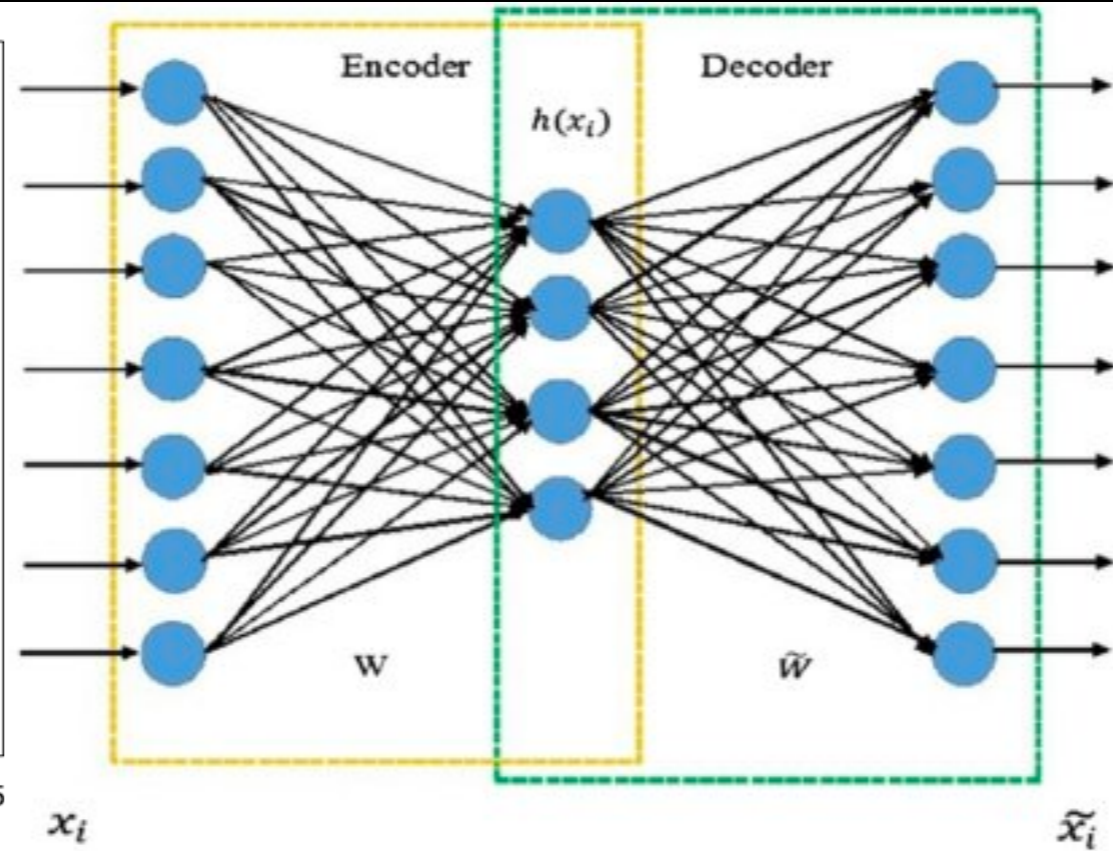
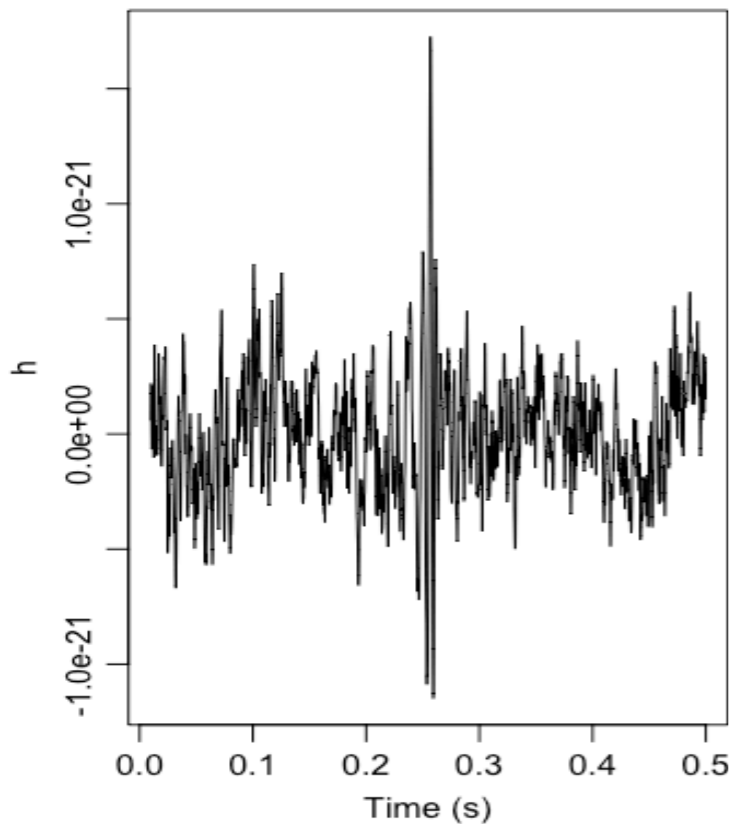




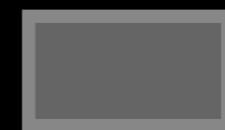
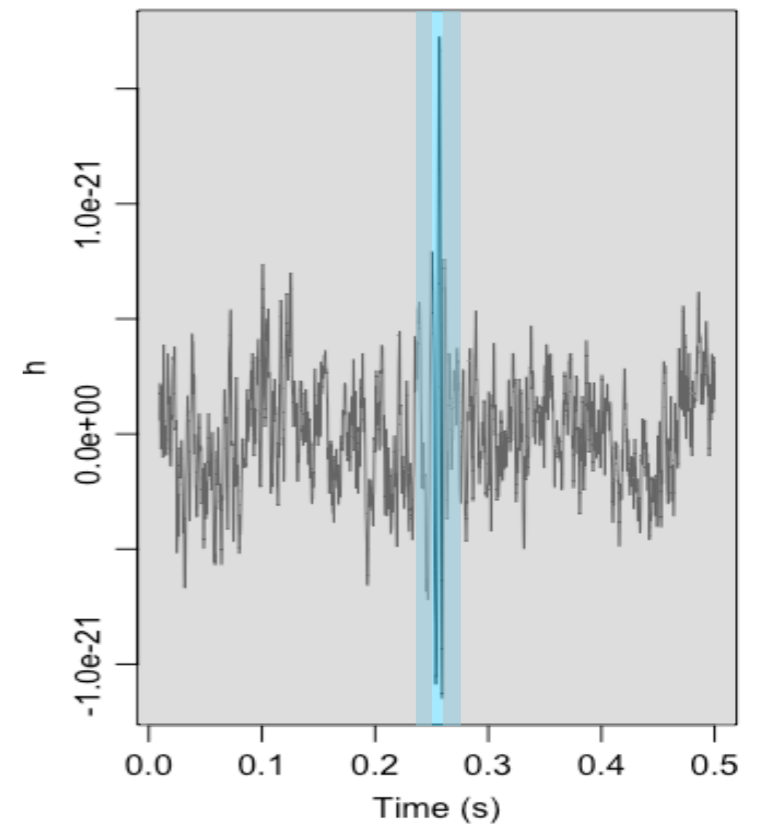
# ON-GOING/FUTURE WORKS

2. Further work will be devoted to improve the performance of anomaly detection with machine learning techniques (e.g. autoencoder).

Input



Output



Normal Abnormal

# ON-GOING/FUTURE WORKS

## 3. Template-free parameter estimation with AR spectral analysis

- Cleaned signal can be fitted with an 2nd stage AR model.
- Signal can be reconstructed from the best-fit model.
- Characteristic equation can be obtained from  $\{a_j\}$  and the order  $p$ .

$$F(z) = 1 - \sum_{j=1}^p a_j z^j = 0$$

- QNM frequency/Damping can be obtained from the complex roots.

$$z_k = \exp(i2\pi f_k \Delta t)$$

$$\text{Re}(f_k) \longrightarrow \text{Frequency}$$

$$\text{Im}(f_k) \longrightarrow \text{Damping}$$

It has been shown that this can extract the ring-down freq./damping timescale from GW150914 (Shinkai 2018,2019).

THANK YOU VERY MUCH